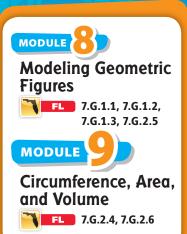
Geometry



CAREERS IN MATH

Product Design Engineer A product design engineer works to design and develop manufactured products and equipment. A product design engineer uses math to design and modify models, and to calculate costs in producing their designs.

If you are interested in a career in product design engineering, you should study these mathematical subjects:

- Algebra
- Geometry
- Trigonometry
- Statistics
- Calculus

Research other careers that require the use of mathematics to design and modify products.

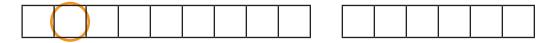
Unit 4 Performance Task

At the end of the unit, check out how **product design engineers** use math.

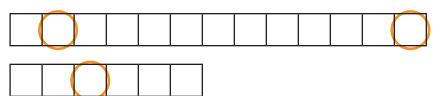


Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters to answer the riddle at the bottom of the page.

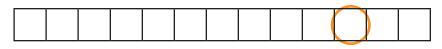
1. NEONGTURC LANSEG



2. LEOTECRAYMPMN SEGLAN



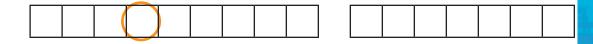
3. RIMEUCEEFNRCC



4. LEARATL ERAA



5. PIECOTMOS GUISEFR



- 1. Angles that have the same measure. (Lesson 8.4)
- 2. Two angles whose measures have a sum of 90 degrees. (Lesson 8.4)
- 3. The distance around a circle. (Lesson 9.1)
- 4. The sum of the areas of the lateral faces of a prism. (Lesson 9.4)
- 5. A two-dimensional figure made from two or more geometric figures. (Lesson 9.3)

Q: What do you say when you see an empty parrot cage?

A: _____

Modeling Geometric Figures



ESSENTIAL QUESTION

How can you use proportions to solve real-world geometry problems?



LESSON 8.1 Similar Shapes and Scale Drawings

LESSON 8.2 Geometric Drawings





.ross Sections

LESSON 8.4 Angle Relationships

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Real-World Video

Architects make blueprints and models of their designs to show clients and contractors. These scale drawings and scale models have measurements in proportion to those of the project when built.





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Animated Math

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Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are Ready?

Complete these exercises to review skills you will need for this module.



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Solve Two-Step Equations

EXAMPLE 5x + 3 = -75x + 3 - 3 = -7 - 3Subtract 3 from both sides. 5x = -10Simplify. $\frac{5x}{5} = \frac{-10}{5}$ Divide both sides by 5. x = -2

Solve.

1.	3x + 4 = 10	2.	5x - 11 = 34	3.	-2x + 5 = -9	4.	-11 = 8x + 13
5.	4x-7=-27	6.	$\frac{1}{2}x + 16 = 39$	7.	12 = 2x - 16	8.	5x - 15 = -65

Solve Proportions

EXAMPLE	$\frac{a}{4} = \frac{27}{18}$	
	$a \times 18 = 4 \times 27$	Write the cross products.
	18 <i>a</i> = 108	Simplify.
	$\frac{18a}{18} = \frac{108}{18}$	Divide both sides by 18.
	<i>a</i> = 6	

Solve for *x*.

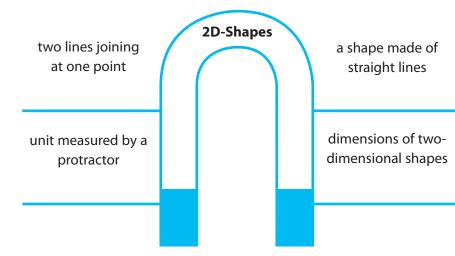
9.	$\frac{x}{5} = \frac{18}{30}$	10. $\frac{x}{12} = \frac{24}{36}$	11. $\frac{3}{9} = \frac{x}{3}$	12.	$\frac{14}{15} = \frac{x}{75}$
13.	$\frac{8}{x} = \frac{14}{7}$	14. $\frac{14}{x} = \frac{2}{5}$	15. $\frac{5}{6} = \frac{x}{15}$	16.	$\frac{81}{33} = \frac{x}{5.5}$

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Visualize Vocabulary

Use the 🖌 words to complete the graphic. You may put more than one word on each line.



Understand Vocabulary

Complete each sentence using a preview word.

1. What is a proportional two-dimensional drawing of an object?



3. _____ are angles whose measures have a

sum of 90°.

Vocabulary

Review Words

- ✓ angle (ángulo)
- ✓ degree (grado) dimension (dimensión)
- length (longitud)
 proportion (proporción)
- ✓ polygon (polígono) ratio (razón)
- ✓ width (ancho)

Preview Words

adjacent angles (ángulos adyacentes) complementary angles (ángulos complementarios) congruent angles (ángulos congruentes) cross section (sección transversal) intersection (intersección) scale (escala) scale drawing (dibujo a escala) supplementary angles (ángulos suplementarios) vertical angles (ángulos *verticales*)

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write each highlighted vocabulary word on one side of a flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions in this module.



MODULE 8 Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

7.G.1.1

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Key Vocabulary

scale (escala)

The ratio between two sets of measurements.

What It Means to You

You will learn how to calculate actual measurements from a scale drawing.

UNPACKING EXAMPLE 7.G.1.1

A photograph of a painting has dimensions 5.4 cm and 4 cm. The scale factor is $\frac{1}{15}$. Find the length and width of the actual painting.

$\frac{1}{15} = \frac{5.4}{\ell}$	$\frac{1}{15} = \frac{4}{W}$
$\frac{1\times5.4}{15\times5.4} = \frac{5.4}{\ell}$	$\frac{1\times4}{15\times4}=\frac{4}{w}$
$15 \times 5.4 = \ell$	$15 \times 4 = w$
81 = <i>l</i>	60 = <i>w</i>

The painting is 81 cm long and 60 cm wide.

]_=__ 7.G.2.5

Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.

Key Vocabulary

supplementary angles

(ángulos suplementarios)

Two angles whose measures have a sum of 180° .



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What It Means to You

You will learn about supplementary, complementary, vertical, and adjacent angles. You will solve simple equations to find the measure of an unknown angle in a figure.

UNPACKING EXAMPLE 7.G.2.5

Suppose $m \angle 1 = 55^{\circ}$.

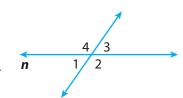
Adjacent angles formed by two intersecting lines are supplementary.

 $m \angle 1 + m \angle 2 = 180^{\circ}$

 $55^{\circ} + m \angle 2 = 180^{\circ}$ Substitute.

 $m\angle 2 = 180^{\circ} - 55^{\circ}$

$$= 125^{\circ}$$



Similar Shapes and Scale Drawings



Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

ESSENTIAL QUESTION

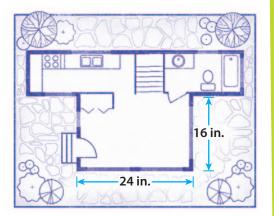
How can you use scale drawings to solve problems?



Finding Dimensions

Scale drawings and scale models are used in mapmaking, construction, and other trades.

A blueprint is a technical drawing that usually displays architectural plans. Pete's blueprint shows a layout of a house. Every 4 inches in the blueprint represents 3 feet of the actual house. One of the walls in the blueprint is 24 inches long. What is the actual length of the wall?





A Complete the table to find the actual length of the wall.

Blueprint length (in.)	4	8	12	16	20	24
Actual length (ft)	3	б				

Reflect

- 1. In Pete's blueprint the length of a side wall is 16 inches. Find the actual length of the wall.
- **2.** The back wall of the house is 33 feet long. What is the length of the back wall in the blueprint?
- **3.** Check for Reasonableness How do you know your answer to **2** is reasonable?

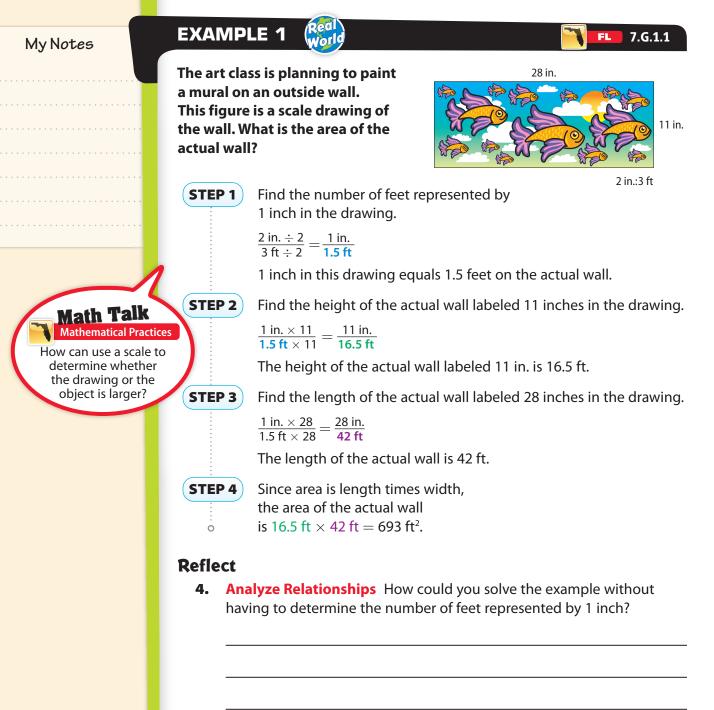


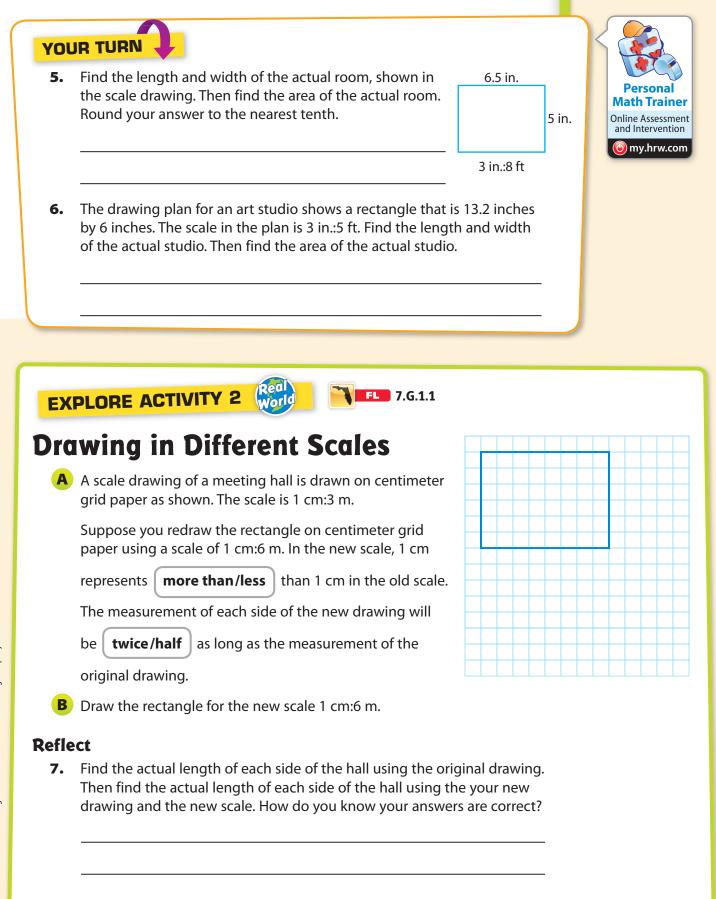
Using a Scale Drawing to Find Area

A **scale drawing** is a proportional two-dimensional drawing of an object. Scale drawings can represent objects that are smaller or larger than the actual object.

A **scale** is a ratio between 2 sets of measurements. It shows how a dimension in a scale drawing is related to the actual object. Scales are usually shown as two numbers separated by a colon such as 1:20 or 1 cm:1 m. Scales can be shown in the same unit or in different units.

You can solve scale-drawing problems by using proportional reasoning.





Guided Practice

1. The scale of a room in a blueprint is 3 in : 5 ft. A wall in the same blueprint is 18 in. Complete the table. (Explore Activity 1)

How long is the actual wall? A window in the room has an actual width of 2.5 feet. Find the width of the window in the blueprint		3								
A window in the room has an actual width of 2.5 feet. Find the width of the window in the blueprint. The scale in the drawing is 2 in. : 4 ft. What are the length and width of the actual room? Find the area of the actual room. 14 in. (Example 1) 7 in. 7 in. 7 in. (Fhe scale in the drawing is 2 cm : 5 m. What are the length and width of the actual room? Find the area of the actual room. (Example 1) 10 cm 6 cm 6 cm A scale drawing of a cafeteria is drawn on centimeter grid paper as shown. The scale is 1 cm : 4 m. (Explore Activity 2)	Actual length (ft)									
b. A window in the room has an actual width of 2.5 feet. Find the width of the window in the blueprint. The scale in the drawing is 2 in. : 4 ft. What are the length and uidth of the actual room? Find the area of the actual room. (Example 1) 7 in. The scale in the drawing is 2 cm : 5 m. What are the length and width of the actual room? Find the area of the actual room. (Example 1) 6 cm A scale drawing of a cafeteria is drawn on centimeter grid paper as shown. The scale is 1 cm : 4 m. (Explore Activity 2)	Actual length (It)									
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width of the actual room? Find the area of the actual room. (Example 1) 7 in.						t				
7 in. 7 in. The scale in the drawing is 2 cm : 5 m. What are the length and width of the actual room? Find the area of the actual room. (Example 1) 10 cm A scale drawing of a cafeteria is drawn on centimeter grid paper as shown. The scale is 1 cm : 4 m. (Explore Activity 2) 6 cm	width of the actual ro	-			-		[14 in.	
of the actual room? Find the area of the actual room. (Example 1) 6 cm A scale drawing of a cafeteria is drawn on centimeter grid paper as shown. The scale is 1 cm : 4 m. (Explore Activity 2)	Example 1)						7 in.			
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6 cm A scale drawing of a cafeteria is drawn on centimeter grid paper as shown. The scale is 1 cm : 4 m. (Explore Activity 2)	The scale in the drawir	ng is 2 cm : 5 r	m. What a	ire the	lengtł	n and v	- ⁻ vidth		10 c	m
A scale drawing of a cafeteria is drawn on centimeter grid paper as shown. The scale is 1 cm : 4 m. (Explore Activity 2)	of the actual room? Fir	nd the area of	the actua	al room	n. <mark>(</mark> Exa	mple	1)			
shown. The scale is 1 cm : 4 m. (Explore Activity 2)								6 cm		
	-				-	d pap	er as			
						a scale	of 1 cr	n:6 m.		
			g.re.							
	scale? What are the	actual dimen	sions of tl	ne cafe	teria u	sing th	ie new	scale?		
scale? What are the actual dimensions of the cafeteria using the new scale?										
scale? What are the actual dimensions of the cafeteria using the new scale?										
	SSENTIAL QUES	TION CHEC	K-IN							
scale? What are the actual dimensions of the cafeteria using the new scale?	f you have an accurat	o complete c	calo drav	ving ar	nd tha	ccalo	which			

6

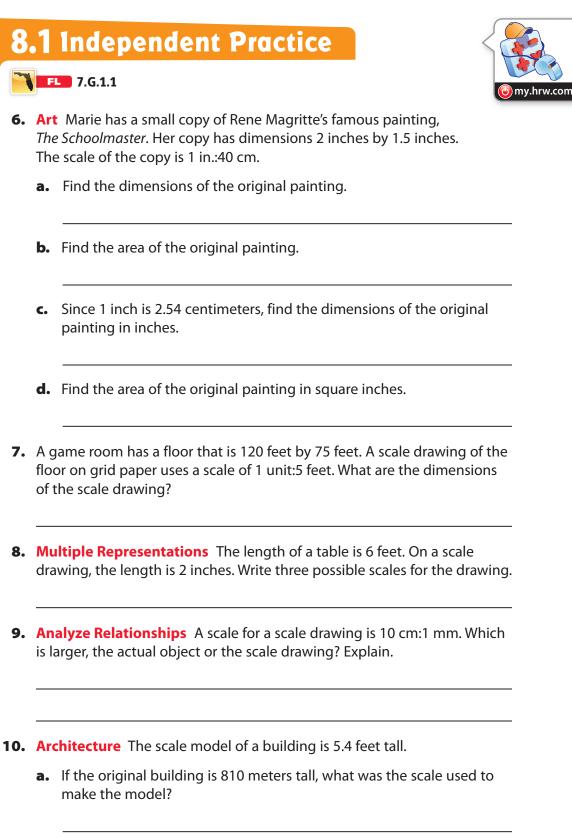
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b. If the model is made out of tiny bricks each measuring 0.4 inch in height, how many bricks tall is the model?

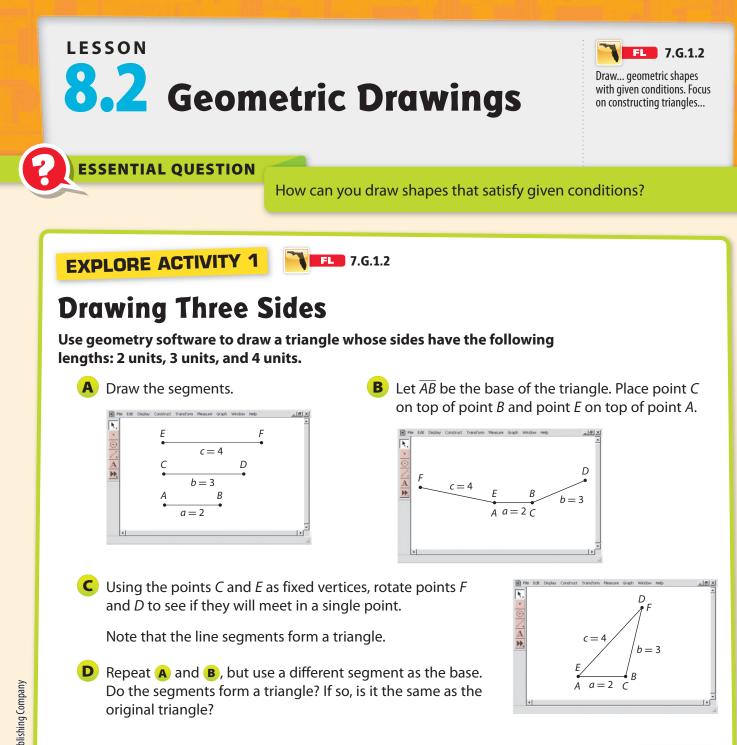
- **11.** You have been asked to build a scale model of your school out of toothpicks. Imagine your school is 30 feet tall. Your scale is 1 ft:1.26 cm.
 - **a.** If a toothpick is 6.3 cm tall, how many toothpicks tall will your model be?
 - **b.** Your mother is out of toothpicks, and suggests you use cotton swabs instead. You measure them, and they are 7.6 cm tall. How many cotton swabs tall will your model be?



- **12. Draw Conclusions** The area of a square floor on a scale drawing is 100 square centimeters, and the scale of the drawing is 1 cm : 2 ft. What is the area of the actual floor? What is the ratio of the area in the drawing to the actual area?
- **13.** Multiple Representations Describe how to redraw a scale drawing with a new scale.

Work Area

14. Represent Real-World Problems Describe how several jobs or professions might use scale drawings at work.



E Use geometry software to draw a triangle with sides of length 2, 3, and 6 units, and one with sides of length 2, 3, and 5 units. Do the line segments form triangles? How does the sum of the lengths of the two shorter sides of each triangle compare to the length of the third side?



Reflect

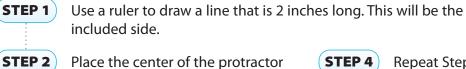
1. **Conjecture** Do two segments of lengths *a* and *b* units and a longer segment of length *c* units form one triangle, more than one, or none?

Two Angles and Their Included Side

Use a ruler and a protractor to draw each triangle.

Triangle 1	Triangle 2
Angles: 30° and 80°	Angles: 55° and 50°
Length of included side: 2 inches	Length of included side: 1 inch

A Draw Triangle 1.



on the left end of the 2-in. line.

Then make a 30°-angle mark.

STEP 4 Repeat Step 2 on the right side of the triangle to construct the 80° angle.

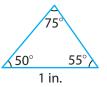
2 հանդեսնուն

STEP 3 Draw a line connecting the left side of the 2-in. line and the 30°-angle mark. This will be the 30° angle.

STEP 5 The side of the 80° angle and the side of the 30° angle will intersect. This is Triangle 1 with angles of 30° and 80° and an included side of 2 inches.



B Use the steps in A to draw Triangle 2.



Ó

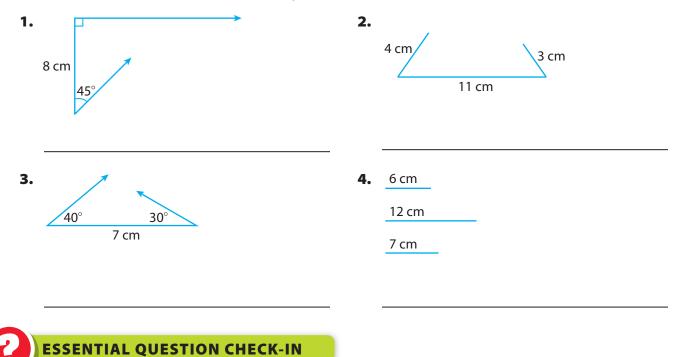
Reflect

2. Conjecture When you are given two angle measures and the length of the included side, do you get a unique triangle?

Class_

Guided Practice

Tell whether each figure creates the conditions to form a unique triangle, more than one triangle, or no triangle. (Explore Activities 1 and 2)



5. Describe lengths of three segments that could **not** be used to form a triangle.

8.2 Independent Practice



- **6.** On a separate piece of paper, try to draw a triangle with side lengths of 3 centimeters and 6 centimeters, and an included angle of 120°. Determine whether the given segments and angle produce a unique triangle, more than one triangle, or no triangle.
- **7.** A landscape architect submitted a design for a triangle-shaped flower garden with side lengths of 21 feet, 37 feet, and 15 feet to a customer. Explain why the architect was not hired to create the flower garden.

8. Make a Conjecture The angles in an actual triangle-shaped traffic sign all have measures of 60°. The angles in a scale drawing of the sign all have measures of 60°. Explain how you can use this information to decide whether three given angle measures can be used to form a unique triangle or more than one triangle.

FOCUS ON HIGHER ORDER THINKING

9. Communicate Mathematical Ideas The figure on the left shows a line segment 2 inches long forming a 45° angle with a dashed line whose length is not given. The figure on the right shows a compass set at a width of $1\frac{1}{2}$ inches with its point on the top end of the 2-inch segment. An arc is drawn intersecting the dashed line twice.

2 in.

45

2 in

45

Explain how you can use this figure to decide whether two sides and an angle **not** included between them can be used to form a unique triangle, more than one triangle, or no triangle.

10. Critical Thinking Two sides of an isosceles triangle have lengths of 6 inches and 15 inches, respectively. Find the length of the third side. Explain your reasoning.





Work Area

8.3 Cross Sections

FL 7.G.1.3 Describe the two-dimensional figures that result from slicing three-dimensional figures ...

ESSENTIAL QUESTION

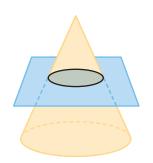
How can you identify cross sections of three-dimensional figures?

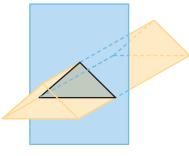
EXPLORE ACTIVITY 1 7.G.1.3

Cross Sections of a Right Rectangular Prism

An **intersection** is a point or set of points common to two or more geometric figures. A **cross section** is the intersection of a three-dimensional figure and a plane. Imagine a plane slicing through the pyramid shown, or through a cone or a prism.





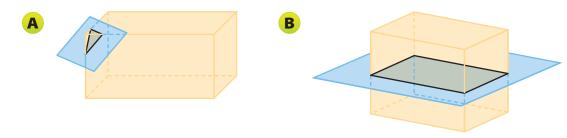


This figure shows the intersection of the cone and a plane. The cross section is a circle.

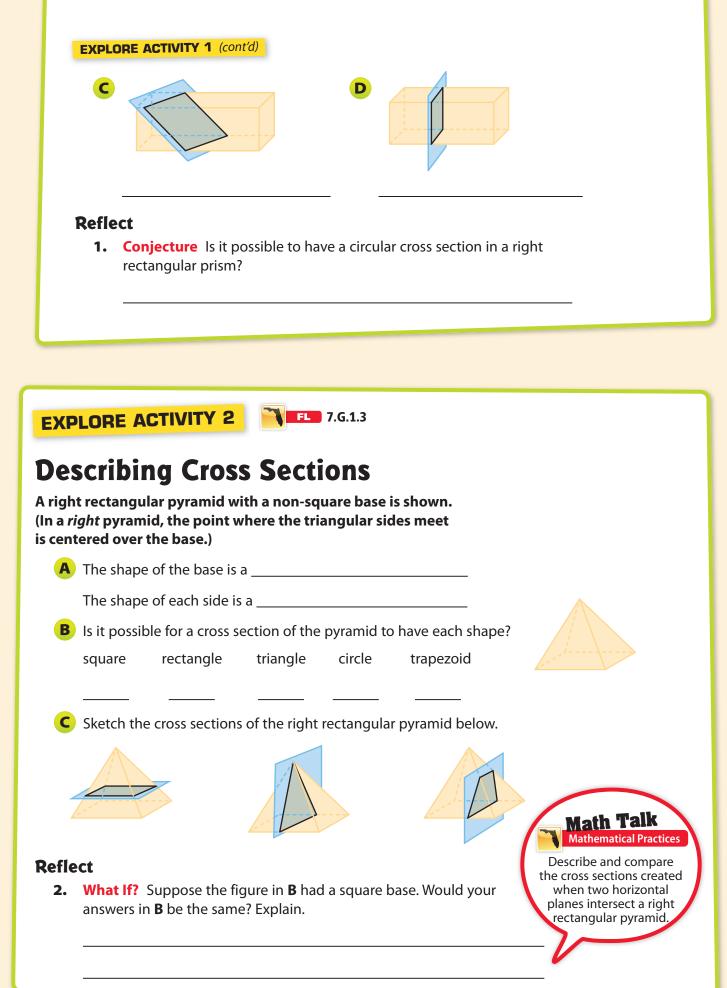
This figure shows the intersection of a triangular prism and a plane. The cross section is a triangle.

A three-dimensional figure can have several different cross sections depending on the position and the direction of the slice. For example, if the intersection of the plane and cone were vertical, the cross section would form a triangle.

Describe each cross section of the right rectangular prism with the name of its shape. (In a *right* prism, all the sides connecting the bases are rectangles at right angles with the base.)



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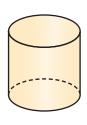


Class_

Guided Practice Describe each cross section. 1. 2. (Explore Activity 1) (Explore Activity 2) 3. 4. h (Explore Activity 2) (Explore Activity 2) **ESSENTIAL QUESTION CHECK-IN** 5. What is the first step in describing what figure results when a given plane intersects a given three-dimensional figure?



6. Describe different ways in which a plane might intersect the cylinder, and the cross section that results.



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7. Make a Conjecture What cross sections might you see when a plane intersects a cone that you would **not** see when a plane intersects a

pyramid or a prism? _____



FOCUS ON HIGHER ORDER THINKING

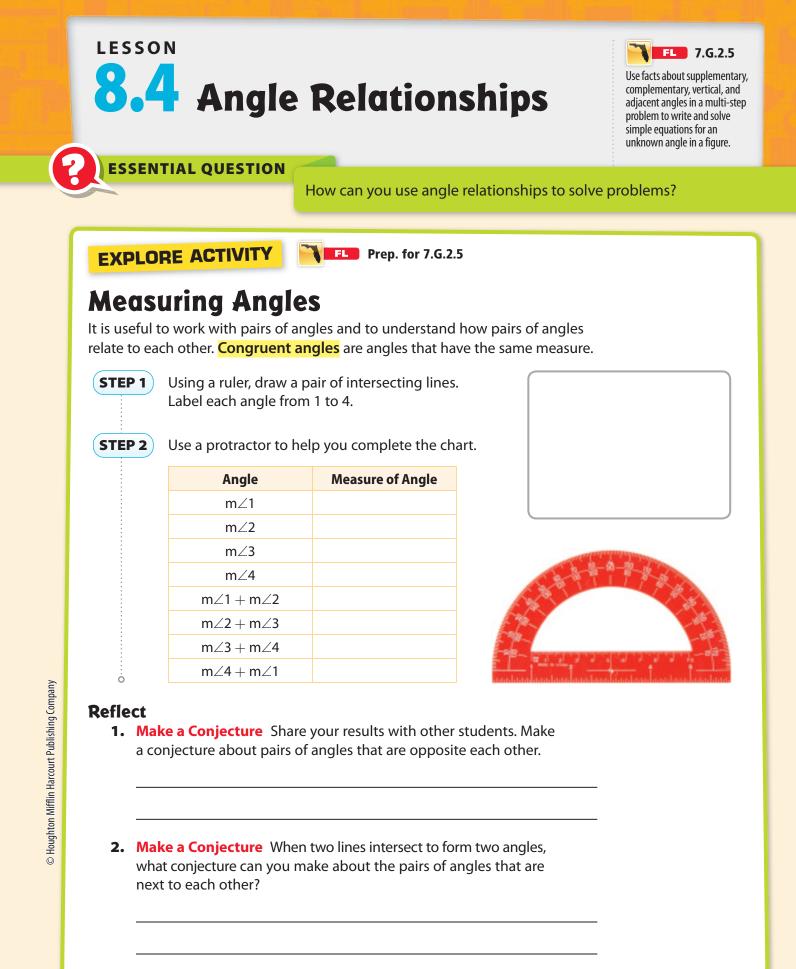
8. Critical Thinking The two figures on the left below show that you can form a cross section of a cube that is a pentagon. Think of a plane cutting the cube at an angle in such a way as to slice through five of the cube's six faces. Draw dotted lines on the third cube to show how to form a cross section that is a hexagon.



- **9.** Analyze Relationships A sphere has a radius of 12 inches. A horizontal plane passes through the center of the sphere.
 - **a.** Describe the cross section formed by the plane and the sphere.
 - **b.** Describe the cross sections formed as the plane intersects the interior of the sphere but moves away from the center.

- **10.** Communicate Mathematical Ideas A right rectangular prism is intersected by a horizontal plane and a vertical plane. The cross section formed by the horizontal plane and the prism is a rectangle with dimensions 8 in. and 12 in. The cross section formed by the vertical plane and the prism is a rectangle with dimensions 5 in. and 8 in. Describe the faces of the prism, including their dimensions. Then find its volume.
- **11. Represent Real-World Problems** Describe a real-world situation that could be represented by planes slicing a three-dimensional figure to form cross sections.

Work Area





Math Talk

Why or why not?

Mathematical Practices Are $\angle BFD$ and $\angle AFE$ vertical angles?

Angle Pairs and One-Step Equations

Vertical angles are the opposite angles formed by two intersecting lines. Vertical angles are congruent because the angles have the same measure.

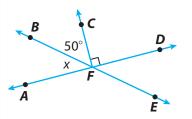
Adjacent angles are pairs of angles that share a vertex and one side but do not overlap.

Complementary angles are two angles whose measures have a sum of 90°.

Supplementary angles are two angles whose measures have a sum of 180°. You discovered in the Explore Activity that adjacent angles formed by two intersecting lines are supplementary.

EXAMPLE 1

Use the diagram.



A Name a pair of vertical angles.

Vertical angles are opposite angles formed by intersecting lines.

 $\angle AFB$ and $\angle DFE$ are vertical angles.

B Name a pair of adjacent angles.

Adjacent angles share a vertex and a side but do not overlap.

 $\angle AFB$ and $\angle BFD$ are adjacent angles.

Name a pair of supplementary angles.

Adjacent angles formed by intersecting lines are supplementary.

 $\angle AFB$ and $\angle BFD$ are supplementary angles.



D Name two pairs of supplementary angles that include $\angle DFE$.

Any angle that forms a line with $\angle DFE$ is a supplementary angle to $\angle DFE$.

 $\angle DFE$ and $\angle EFA$ are supplementary angles, as are $\angle DFE$ and $\angle DFB$.

7.G.2.5

Find the measure of $\angle AFB$.

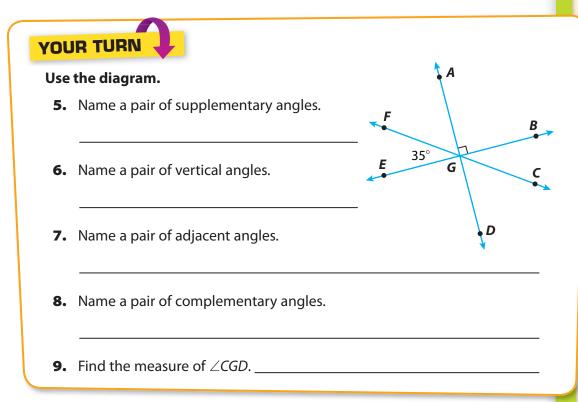
Use the fact that $\angle AFB$ and $\angle BFD$ in the diagram are supplementary angles to find m $\angle AFB$.

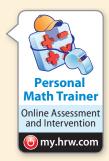
 $m \angle AFB + m \angle BFD = 180^{\circ}$ They are supplementary angles. $x + 140^{\circ} = 180^{\circ}$ $m \angle BFD = 50^{\circ} + 90^{\circ} = 140^{\circ}$ $\underline{-140^{\circ}} - \underline{-140^{\circ}}$ Subtract 140 from both sides. $x = 40^{\circ}$

The measure of $\angle AFB$ is 40°.

Reflect

- **3.** Analyze Relationships What is the relationship between $\angle AFB$ and $\angle BFC$? Explain.
- **4. Draw Conclusions** Are $\angle AFC$ and $\angle BFC$ adjacent angles? Why or why not?





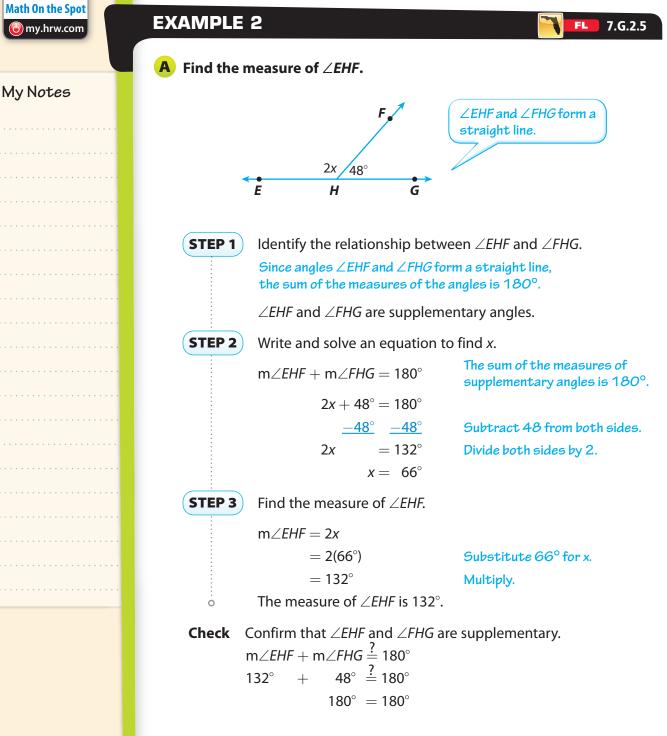
My Notes

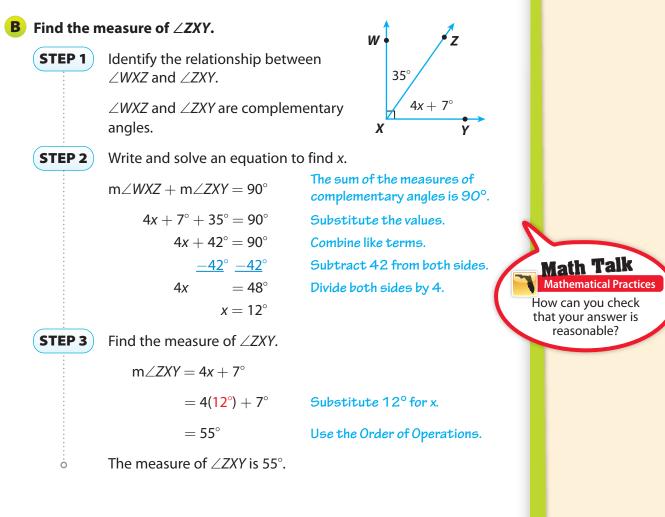


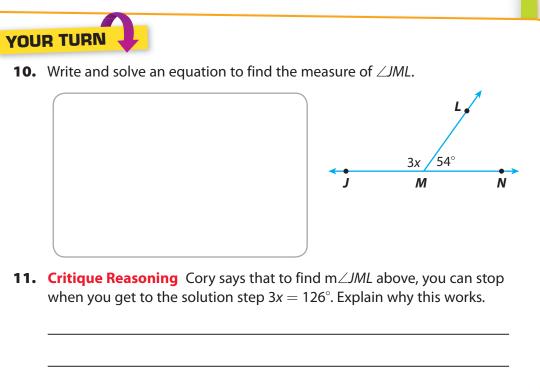


Angle Pairs and Two-Step Equations

Sometimes solving an equation is only the first step in using an angle relationship to solve a problem.

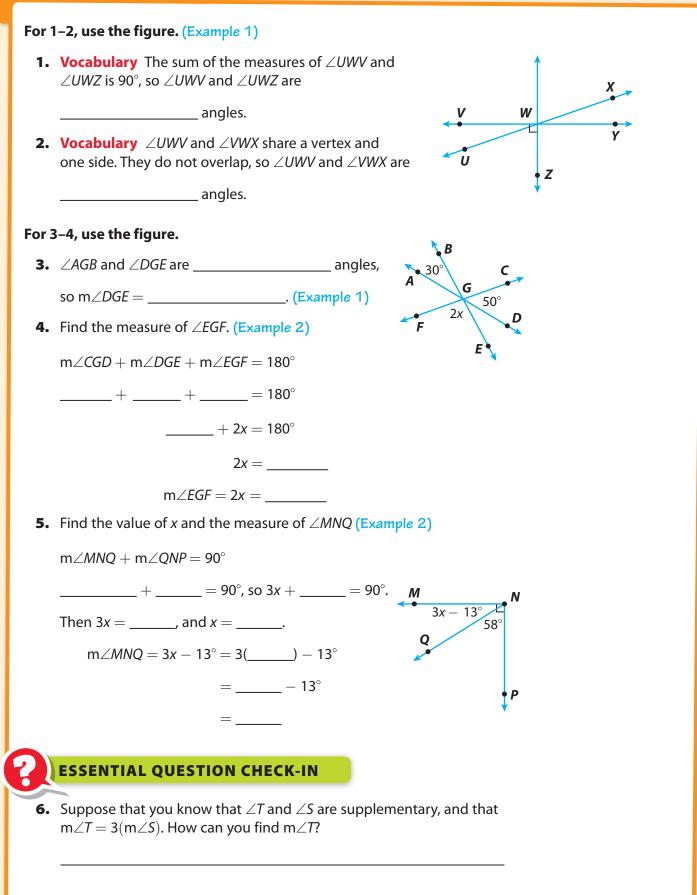






Lesson 8.4 255

Guided Practice

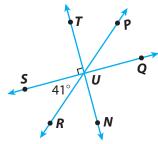


Class_

8.4 Independent Practice



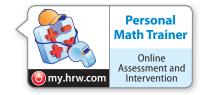
For 7–11, use the figure.



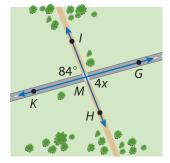
7. Name a pair of adjacent angles. Explain why they are adjacent.

- 8. Name a pair of acute vertical angles.
- **9.** Name a pair of supplementary angles.
- **10.** Justify Reasoning Find m∠QUR. Justify your answer.

11. Draw Conclusions Which is greater, m∠*TUR* or m∠*RUQ*? Explain.

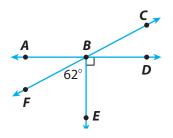


For 12–13, use the figure. A bike path crosses a road as shown. Solve for each indicated angle measure or variable.



- **12.** *x*_____
- **13.** m∠*KMH*_____

For 14–16, use the figure. Solve for each indicated angle measure.

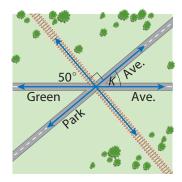


- **14.** m∠*CBE*_____
- **15.** m∠*ABF*_____
- **16.** m∠*CBA*_____
- **17.** The measure of $\angle A$ is 4° greater than the measure of $\angle B$. The two angles are complementary. Find the measure of each angle.
- **18.** The measure of $\angle D$ is 5 times the measure of $\angle E$. The two angles are supplementary. Find the measure of each angle.

19. Astronomy Astronomers sometimes use angle measures divided into degrees, minutes, and seconds. One degree is equal to 60 minutes, and one minute is equal to 60 seconds. Suppose that $\angle J$ and $\angle K$ are complementary, and that the measure of $\angle J$ is 48 degrees, 26 minutes, 8 seconds. What is the measure of $\angle K$?

OT FOCUS ON HIGHER ORDER THINKING

- **20. Represent Real-World Problems** The railroad tracks meet the road as shown. The town will allow a parking lot at angle *K* if the measure of angle *K* is greater than 38°. Can a parking lot be built at angle *K*? Why or why not?
- **21.** Justify Reasoning Kendra says that she can draw $\angle A$ and $\angle B$ so that $m \angle A$ is 119° and $\angle A$ and $\angle B$ are complementary angles. Do you agree or disagree? Explain your reasoning.



Work Area

- **22. Draw Conclusions** If two angles are complementary, each angle is called a *complement* of the other. If two angles are supplementary, each angle is called a *supplement* of the other.
 - **a.** Suppose $m \angle A = 77^{\circ}$. What is the measure of a complement of a complement of $\angle A$? Explain.

b. What conclusion can you draw about a complement of a complement of an angle? Explain.

MODULE QUIZ

Ready to Go On?

8.1 Similar Shapes and Scale Drawings



1. A house blueprint has a scale of 1 in. : 4 ft. The length and width of each room in the actual house are shown in the table. Complete the table by finding the length and width of each room on the blueprint.

	Living room	Kitchen	Office	Bedroom	Bedroom	Bathroom
Actual $\boldsymbol{\ell} imes \boldsymbol{w}$ (ft)	16 × 20	12 × 12	8 × 12	20 × 12	12 × 12	6 × 8
Blueprint $\boldsymbol{\ell} \times \boldsymbol{w}$ (in.)						

8.2 Geometric Drawings

- 2. Can a triangle be formed with the side lengths of 8 cm, 4 cm, and 12 cm?
- **3.** A triangle has side lengths of 11 cm and 9 cm. Which could be the value of

the third side, 20 cm or 15 cm? ______

8.3 Cross Sections

- 4. Name one possible cross section of a sphere.
- 5. Name at least two shapes that are cross sections of a cylinder.

8.4 Angle Relationships

- **6.** \angle *BGC* and \angle *FGE* are _____ angles, so m \angle *FGE* = _____
- **7.** Suppose you know that $\angle S$ and $\angle Y$ are complementary, and

that $m \angle S = 2(m \angle Y) - 30^\circ$. Find $m \angle Y$.

ESSENTIAL QUESTION

8. How can you model geometry figures to solve real-world problems?



MODULE 8 MIXED REVIEW Assessment Readiness



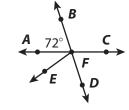
Selected Response

1. Which number can you add to 15 to get a sum of 0?

(▲) −10	© 0
---------	-----

- **B** −15 **D** 15
- 2. Students are painting the backdrop for the school play. The backdrop is 15 feet wide and 10 feet high. Every 16 inches on the scale drawing represents 5 feet on the backdrop. What is the area of the scale drawing?
 - (A) 150 in² (C) 3,096
 - B 6 in²
 D 1,536 in²
- **3.** Two sides of a triangle measure 8 cm and 12 cm. Which of the following CANNOT be the measure of the third side?
 - (A) 4 (C) 8
 - **B** 12 **D** 16
- **4.** A cross section is the intersection of a three-dimensional figure and a _____
 - (A) point (C) line
 - (B) plane (D) set

For 5–6, use the diagram.



- **5.** What is the measure of $\angle BFC$?
 - (A) 18° (C) 72°
 - **B** 108° **D** 144°

- **6.** Which describes the relationship between $\angle BFA$ and $\angle CFD$?
 - (A) adjacent angles
 - (B) complementary angles
 - © supplementary angles
 - **(D)** vertical angles
- **7.** All clothing is being marked down 15%. Which expression represents the new retail price?

(A) 0.85 <i>x</i>	© 1.85 <i>x</i>
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B 1.15*x* **D** 0.15*x*

Mini-Tasks

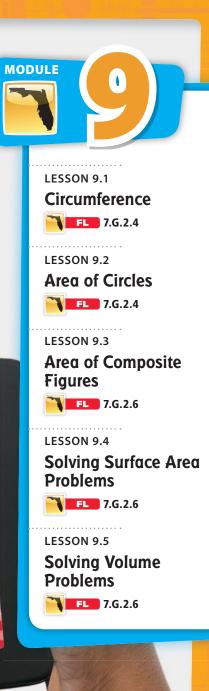
- 8. Ira built a model of the Great Pyramid in Egypt for a school project. The Great Pyramid has a square base with sides of length 756 feet. The height of the Great Pyramid is 481 feet. Ira made his model pyramid using a scale of 1 inch : 20 feet.
 - **a.** What is the length of each side of the base of Ira's pyramid?
 - **b.** What is the area of the base of Ira's pyramid?
 - c. What is the height of Ira's pyramid?
 - **d.** Ira built his model using cross sections that were cut parallel to the base. What shape was each cross section?

Circumference, Area, and Volume



ESSENTIAL QUESTION

How can you apply geometry concepts to solve real-world problems?



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Real-World Video

A 16-inch pizza has a diameter of 16 inches. You can use the diameter to find circumference and area of the pizza. You can also determine how much pizza in one slice of different sizes of pizzas.

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to the online edition,

video tutor, and more.



Animated Math

Interactively explore how math works.



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Get immediate feedback and help as you work through practice sets.

key concepts to see



Are **VOU** Ready?

Complete these exercises to review skills you will need for this module.



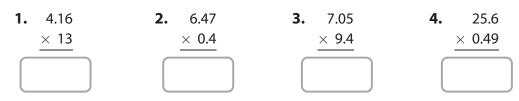
Math Trainer Online Assessment and Intervention

Personal

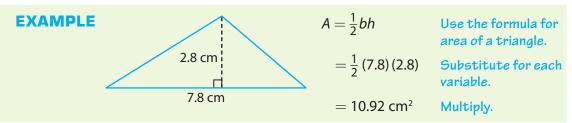
Multiply with Fractions and Decimals

EXAMPLE	7.3 <u>× 2.4</u> 2 9 2	Multiply as you would with whole numbers. Count the total number of decimal places in the two factors.
	<u>+ 1 4 6</u> 1 7.5 2	Place the decimal point in the product so that there are the same number of digits after the decimal point.

Multiply.



Area of Squares, Rectangles, and Triangles



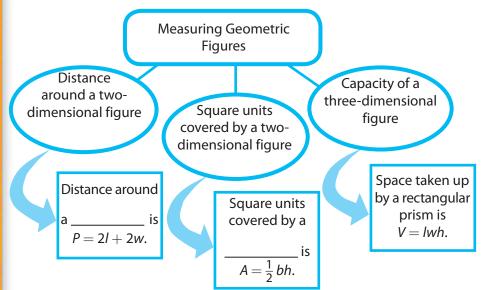
Find the area of each figure.

- 5. triangle with base 14 in. and height 10 in.
- 6. square with sides of 3.5 ft
- **7.** rectangle with length $8\frac{1}{2}$ in. and width 6 in.
- 8. triangle with base 12.5 m and height 2.4 m

Reading Start-Up

Visualize Vocabulary

Use the 🗸 words to complete the graphic. You will put one word in each oval. Then write examples of formulas in each rectangle.



Vocabulary

Review Words

- ✓ area (área) parallelogram (paralelogramo)
- perimeter (perímetro) prism (prisma) rectangle (rectángulo) square (cuadrado) trapezoid (trapecio) triangle (triángulo)
- ✓ volume (volumen)

Preview Words

circumference (*circunferencia*) composite figure (*figura compuesta*) diameter (*diámetro*) radius (*radio*)

Understand Vocabulary

Match the term on the left to the correct expression on the right.

1 circumference A	A. A line segment that passes through the center of a circle and has endpoints on the circle, or the length of that segment.
2. diameter B	3. A line segment with one endpoint at the center of the circle and the other on the circle, or the length of that segment.
3. radius	The distance around a circle.

Active Reading

Four-Corner Fold Before beginning the module, create a four-corner fold to help you organize what you learn. As you study this module, note important ideas, such as vocabulary, properties, and formulas, on the flaps. Use one flap each for circumference, area, surface area, and volume. You can use your FoldNote later to study for tests and complete assignments.



MODULE 9 Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

Net 7.G.2.4

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Key Vocabulary

circumference *(circunferencia)* The distance around a circle.

What It Means to You

You will use formulas to solve problems involving the area and circumference of circles.

UNPACKING EXAMPLE 7.G.2.4

Lily is drawing plans for a circular fountain. The diameter of the fountain is 20 feet. What is the approximate circumference?

 $C = \pi d$ $C \approx 3.14 \cdot 20$ Substitute. $C \approx 62.8$

The circumference of the fountain is about 62.8 feet.

٦ === 7.G.2.6

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Key Vocabulary

volume (volumen)

The number of cubic units inside a three-dimensional solid.

surface area (área total)

The sum of the areas of all the surfaces of a threedimensional solid.



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What It Means to You

You will find area, volume and surface area of real-world objects.

UNPACKING EXAMPLE 7.G.2.6

Find the volume and the surface area of a tissue box before the hole is cut in the top.

The tissue box is a right rectangular prism. The base is $4\frac{3}{8}$ in. by $4\frac{3}{8}$ in. and the height is 5 in.

Use the volume and surface area formulas:

B is the area of the base, h is the height of the box, and *P* is the perimeter of the base.





 $= 2 \left(4 \frac{3}{8} \cdot 4 \frac{3}{8} \right) + \left(4 \cdot 4 \frac{3}{8} \right) 5$ $= 125\frac{25}{32}$ in²

The volume is $95\frac{45}{64}$ in³ and the surface area is $125\frac{25}{32}$ in².

9.1 Circumference

Know the formulas for the area and circumference of a circle and use them to solve problems ...

ESSENTIAL QUESTION

How do you find and use the circumference of a circle?

EXPLORE ACTIVITY

FL 7.G.2.4

Exploring Circumference

A circle is a set of points in a plane that are a fixed distance from the center.

A **radius** is a line segment with one endpoint at the center of the circle and the other endpoint on the circle. The length of a radius is called the radius of the circle.

A **diameter** of a circle is a line segment that passes through the center of the circle and whose endpoints lie on the circle. The length of the diameter is twice the length of the radius. The length of a diameter is called the diameter of the circle.

The **circumference** of a circle is the distance around the circle.

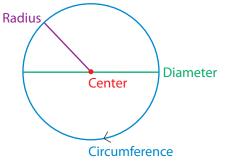
A Use a measuring tape to find the circumference of five circular objects. Then measure the distance across each item to find its diameter. Record the measurements of each object in the table below.

Object	Circumference C	Diameter d	C d

B Divide the circumference of each object by its diameter. Record your answer, rounded to the nearest hundredth, in the table above.

Reflect

1. Make a Conjecture Describe what you notice about the ratio $\frac{C}{d}$ in your table.





Finding Circumference

The ratio of the circumference to the diameter $\frac{C}{d}$ is the same for all circles. This ratio is called π or *pi*, and you can approximate it as 3.14 or as $\frac{22}{7}$. You can use π to find a formula for circumference.

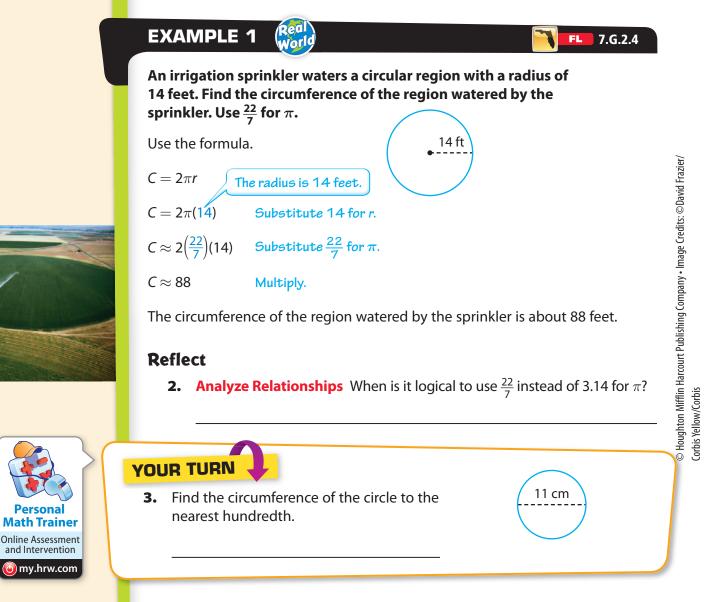
For any circle, $\frac{C}{d} = \pi$. Solve the equation for C to give an equation for the circumference of a circle in terms of the diameter.

 $\frac{C}{d} = \pi$ The ratio of the circumference to the diameter is π . $\frac{C}{d} \times d = \pi \times d$ Multiply both sides by d. $C = \pi d$ Simplify.

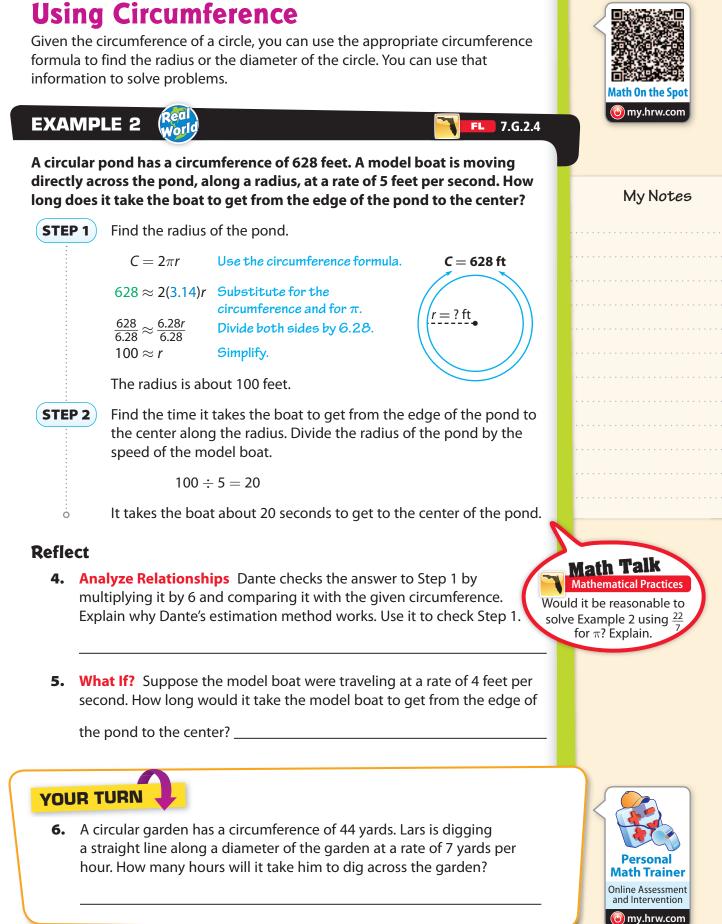
The diameter of a circle is twice the radius. You can use the equation $C = \pi d$ to find a formula for the circumference *C* in terms of the radius *r*.

$$C = \pi d = \pi(2r) = 2\pi r$$

The two equivalent formulas for circumference are $C = \pi d$ and $C = 2\pi r$.

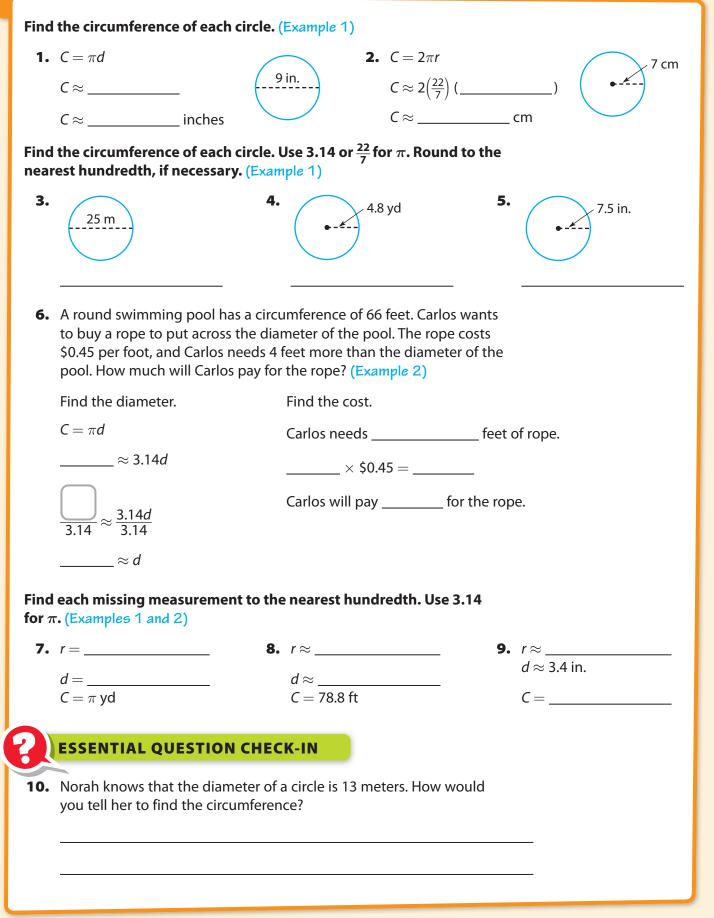


Personal



Lesson 9.1 267

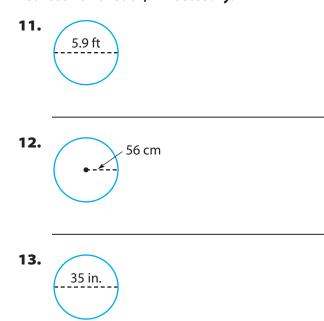
Guided Practice



9.1 Independent Practice

For 11–13, find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π . Round to the nearest hundredth, if necessary.

FL 7.G.2.4



14. In Exercises 11–13, for which problems did you use $\frac{22}{7}$ for π ? Explain your choice.

- **15.** A circular fountain has a radius of 9.4 feet. Find its diameter and circumference to the nearest tenth.
- **16.** Find the radius and circumference of a CD with a diameter of 4.75 inches.
- **17.** A dartboard has a diameter of 18 inches. What are its radius and circumference?



Date



18. Multistep Randy's circular garden has a radius of 1.5 feet. He wants to enclose the garden with edging that costs \$0.75 per foot. About how much will the edging cost? Explain.

Class

19. Represent Real-World Problems The Ferris wheel shown makes 12 revolutions per ride. How far would someone travel during one ride?



20. The diameter of a bicycle wheel is 2 feet. About how many revolutions does the wheel make to travel 2 kilometers? Explain. Hint: 1 km \approx 3,280 ft

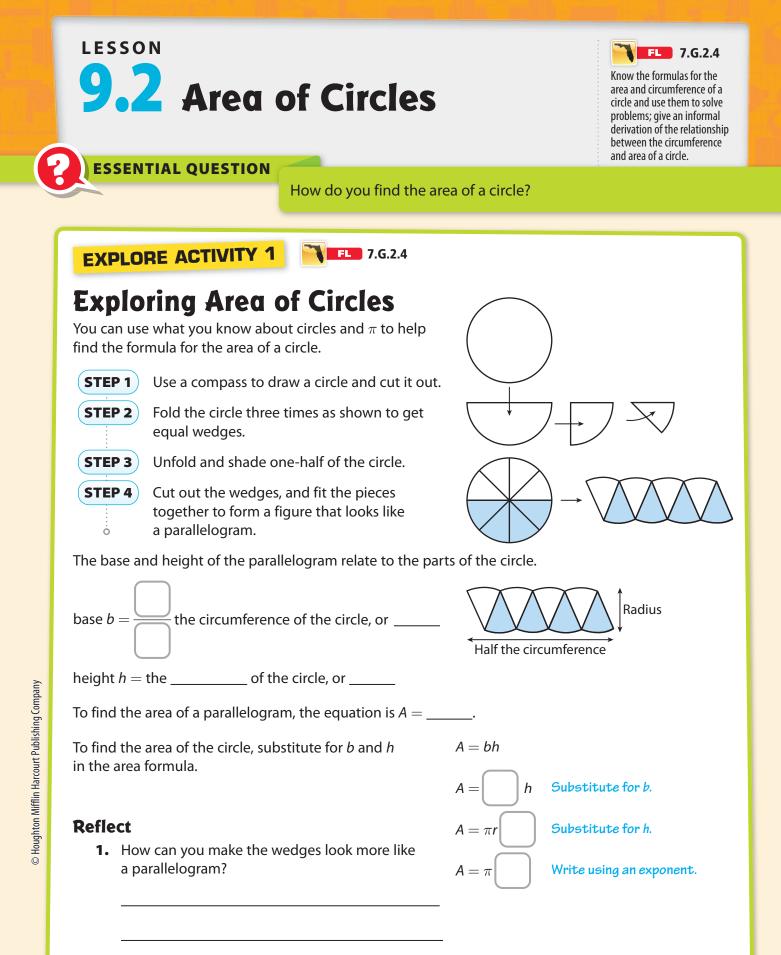
21. Multistep A map of a public park shows a circular pond. There is a bridge along a diameter of the pond that is 0.25 mi long. You walk across the bridge, while your friend walks halfway around the pond to meet you at the other side of the bridge. How much farther does your friend walk? **22.** Architecture The Capitol Rotunda connects the House and the Senate sides of the U.S. Capitol. Complete the table. Round your answers to the nearest foot.

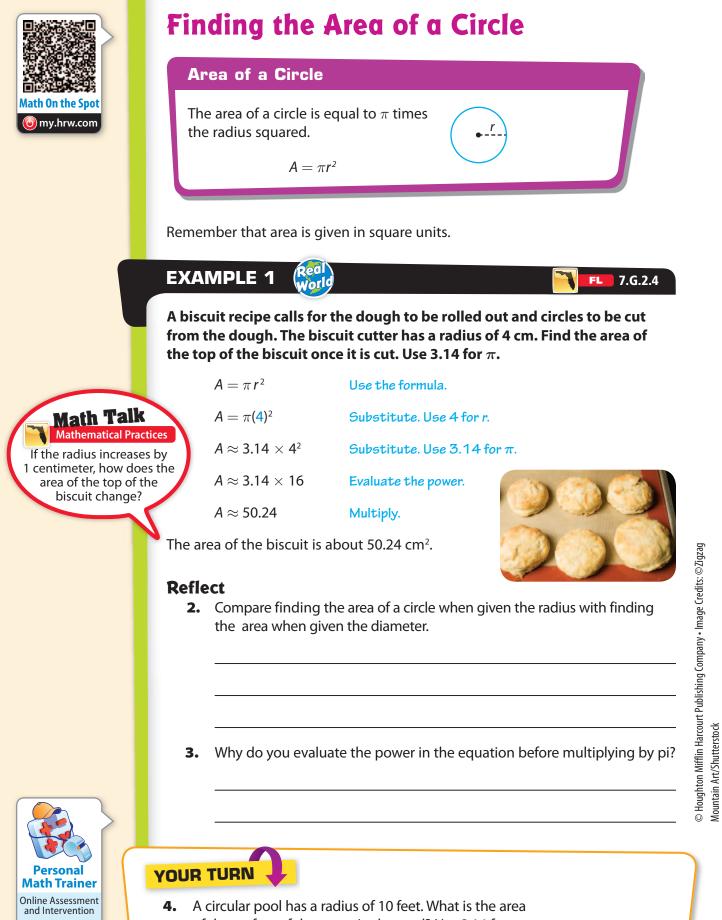
Capitol Rotunda Dimensions	
Height	180 ft
Circumference	301.5 ft
Radius	
Diameter	

FOCUS ON HIGHER ORDER THINKING

- **23. Multistep** A museum groundskeeper is creating a semicircular statuary garden with a diameter of 30 feet. There will be a fence around the garden. The fencing costs \$9.25 per linear foot. About how much will the fencing cost altogether?
- **24.** Critical Thinking Sam is placing rope lights around the edge of a circular patio with a diameter of 18 feet. The lights come in lengths of 54 inches. How many strands of lights does he need to surround the patio edge?
- **25. Represent Real-World Problems** A circular path 2 feet wide has an inner diameter of 150 feet. How much farther is it around the outer edge of the path than around the inner edge?
- **26.** Critique Reasoning A gear on a bicycle has the shape of a circle. One gear has a diameter of 4 inches, and a smaller one has a diameter of 2 inches. Justin says that the circumference of the larger gear is 2 inches more than the circumference of the smaller gear. Do you agree? Explain your answer.

27. Persevere in Problem Solving Consider two circular swimming pools. Pool A has a radius of 12 feet, and Pool B has a diameter of 7.5 meters. Which pool has a greater circumference? How much greater? Justify your answers. Work Area





of the *surface* of the water in the pool? Use 3.14 for π .

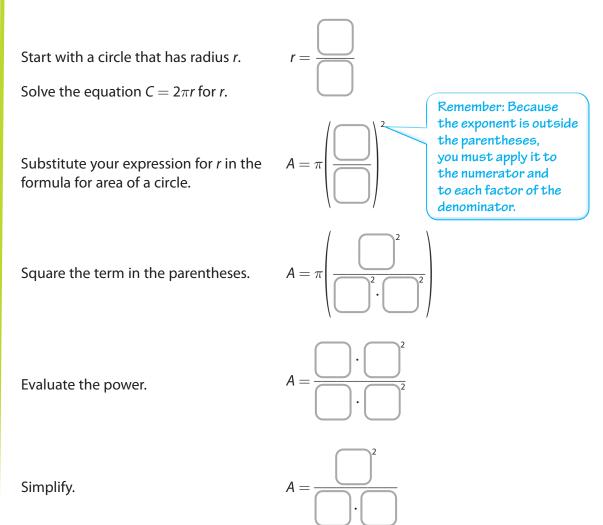
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Finding the Relationship between Circumference and Area

You can use what you know about circumference and area of circles to find a relationship between them.

Find the relationship between the circumference and area of a circle.





Solve for C^2 .

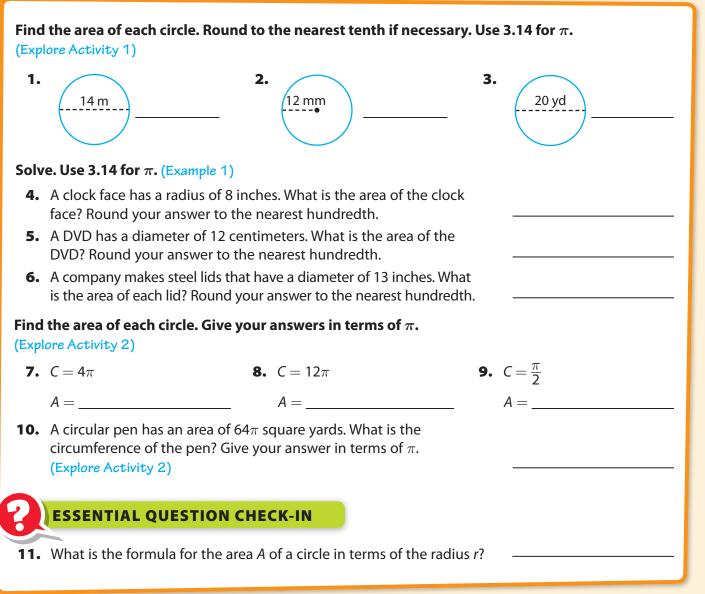
The circumference of the circle squared is equal to

EXPLORE ACTIVITY 2 (cont'd)

Reflect

5. Does this formula work for a circle with a radius of 3 inches? Show your work.

Guided Practice



9.2 Independent Practice

FL 7.G.2.4

- **12.** The most popular pizza at Pavone's Pizza is the 10-inch personal pizza with one topping. What is the area of a pizza with a diameter of 10 inches? Round your answer to the nearest hundredth.
- **13.** A hubcap has a radius of 16 centimeters. What is the area of the hubcap? Round your answer to the nearest hundredth.



- **14.** A stained glass window is shaped like a semicircle. The bottom edge of the window is 36 inches long. What is the area of the stained glass window? Round your answer to the nearest hundredth.
- **15.** Analyze Relationships The point (3, 0) lies on a circle with the center at the origin. What is the area of the circle to the nearest hundredth?
- **16. Multistep** A radio station broadcasts a signal over an area with a radius of 50 miles. The station can relay the signal and broadcast over an area with a radius of 75 miles. How much greater is the area of the broadcast region when the signal is relayed? Round your answer to the nearest square mile.





- 17. Multistep The sides of a square field are 12 meters. A sprinkler in the center of the field sprays a circular area with a diameter that corresponds to a side of the field. How much of the field is **not** reached by the sprinkler? Round your answer to the nearest hundredth.
- **18.** Justify Reasoning A small silver dollar pancake served at a restaurant has a circumference of 2π inches. A regular pancake has a circumference of 4π inches. Is the area of the regular pancake twice the area of the silver dollar pancake? Explain.

19. Analyze Relationships A bakery offers a small circular cake with a diameter of 8 inches. It also offers a large circular cake with a diameter of 24 inches. Does the top of the large cake have three times the area of that of the small cake? If not, how much greater is its area? Explain.

20. Communicate Mathematical Ideas You can use the formula $A = \frac{C^2}{4\pi}$ to find the area of a circle given the circumference. Describe another way to find the area of a circle when given the circumference.

21.	Draw Conclusions Mark wants to order a pizza. Which is the better deal? Explain.	Donnie's	Pizza Pala	ce
		Diameter (in.)	12	18
		Cost (\$)	10	20
22.	Multistep A bear was seen near a campground. Searcher dispatched to the region to find the bear.	s were		
	a. Assume the bear can walk in any direction at a rate of hour. Suppose the bear was last seen 4 hours ago. Ho must the searchers cover? Use 3.14 for π . Round your	w large an area		
	nearest square mile.			
	b. What If? How much additional area would the search	ers have to		
	cover if the bear were last seen 5 hours ago?			
H.	FOCUS ON HIGHER ORDER THINKING		١	Nork Area
23.	Analyze Relationships Two circles have the same radius. Is the combined area of the two circles the same as the area of a circle with twice the radius? Explain.			
24.	Look for a Pattern How does the area of a circle change multiplied by a factor of <i>n</i> , where <i>n</i> is a whole number?	if the radius is		
25.	Represent Real World Problems The bull's-eye on a targ diameter of 3 inches. The whole target has a diameter of 1 part of the whole target is the bull's-eye? Explain.	(

	9.3 Area of Composite Figures	FL 7.G.2.6 Solve real-world and mathematical problems involving area, of objects composed of triangles, quadrilaterals, polygons,
	ESSENTIAL QUESTION How do you find the area of composition	te figures?
	EXPLORE ACTIVITY Real Total 7.G.2.6	
	Exploring Areas of Composite Figures Aaron was plotting the shape of his garden on grid paper. While it was an irregular shape, it was perfect for his yard. Each square on the grid represents 1 square meter. A Describe one way you can find the area of this garden.	
	 B The area of the garden is square meters. C Compare your results with other students. What other methods were used to find the area? 	
Houghton Mifflin Harcourt Publishing Company	D How does the area you found compare with the area found using different methods?	
© Houghton Mif	Reflect 1. Use dotted lines to show two different ways Aaron's garden could be divided up into simple geometric figures.	



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Finding the Area of a Composite Figure

A composite figure is made up of simple geometric shapes. To find the area of a composite figure or other irregular-shaped figure, divide it into simple, nonoverlapping figures. Find the area of each simpler figure, and then add the areas together to find the total area of the composite figure.

Use the chart below to review some common area formulas.

Shape	Area Formula
triangle	$A = \frac{1}{2}bh$
square	$A = s^2$
rectangle	$A = \ell w$
parallelogram	A = bh
trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$

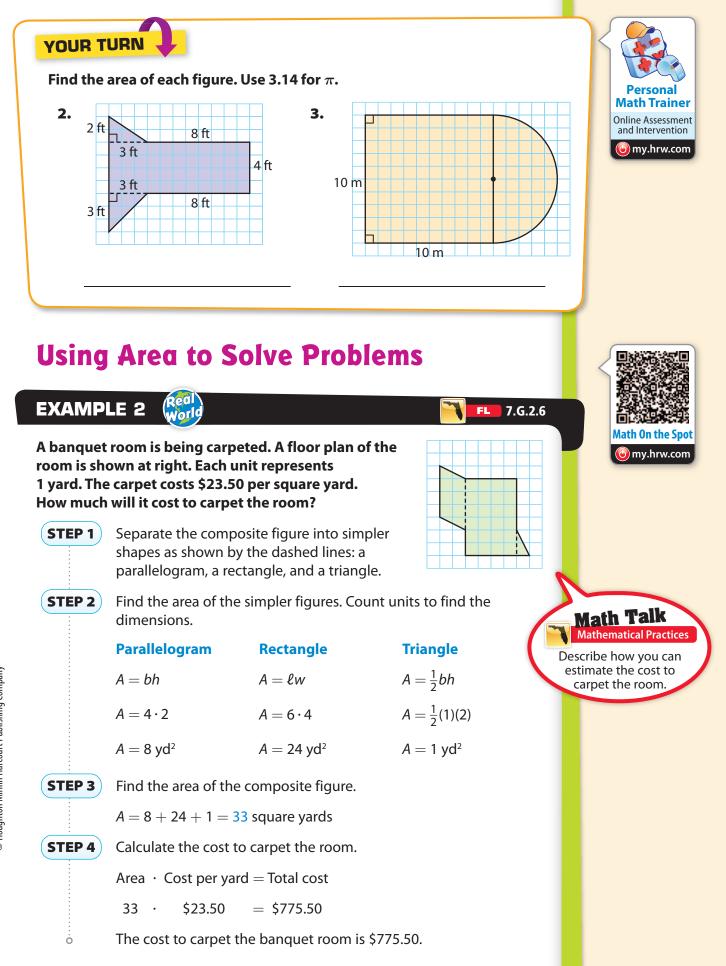
7.G.2.6

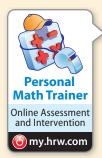
EXAMPLE 1 Real

Find the area of the figure.

STEP 1 Separate the figure into smaller, 10 cm 3 cm familiar figures: a parallelogram 1.5 cm and a trapezoid. 2 cm 4 cm 1.5 cm **STEP 2** Find the area of each shape. 7 cm Area of the Parallelogram Area of the Trapezoid 10 cm 2 cm 3 cm 1.5 cm 1.5 cm 4 cm 7 cm C Houghton Mifflin Harcourt Publishing Company $base_1 = 7 \text{ cm} \quad base_2 = 10 \text{ cm}$ base = 10 cmheight = 1.5 cm height 1.5 cm The top base of the Use the formula. Use the formula. trapezoid is 10 cm $A = \frac{1}{2}h(b_1 + b_2)$ since it is the same A = bhlength as the base of $A = \frac{1}{2} (1.5)(7 + 10)$ the parallelogram. $A = 10 \cdot 1.5$ $A = \frac{1}{2}(1.5)(17) = 12.75$ A = 15The area of the The area of the trapezoid is 12.75 cm². parallelogram is 15 cm². Add the areas to find the total area. **STEP 3** ò $A = 15 + 12.75 = 27.75 \text{ cm}^2$ The area of the figure is 27.75 cm².

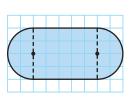
278 Unit 4



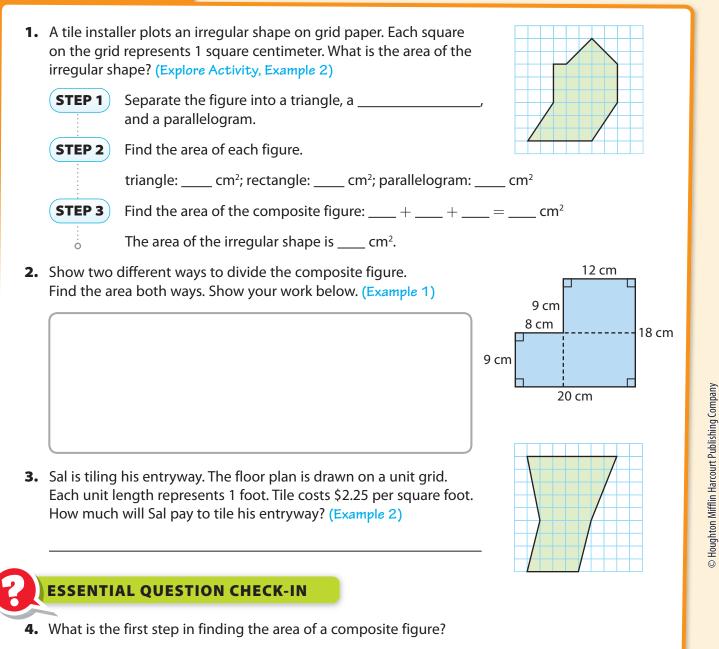




4. A window is being replaced with tinted glass. The plan at the right shows the design of the window. Each unit length represents 1 foot. The glass costs \$28 per square foot. How much will it cost to replace the glass? Use 3.14 for π .



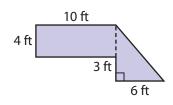
Guided Practice



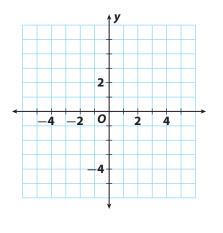
9.3 Independent Practice

FL 7.G.2.6

- 5. A banner is made of a square and a semicircle. The square has side lengths of 26 inches. One side of the square is also the diameter of the semicircle. What is the total area of the banner? Use 3.14 for π .
- 6. Multistep Erin wants to carpet the floor of her closet. A floor plan of the closet is shown.



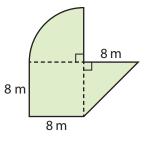
- a. How much carpet does Erin need?
- **b.** The carpet Erin has chosen costs \$2.50 per square foot. How much will it cost her to carpet the floor?
- 7. Multiple Representations Hexagon ABCDEF has vertices A(-2, 4), B(0, 4), C(2, 1), *D*(5, 1), *E*(5, −2), and *F*(−2, −2). Sketch the figure on a coordinate plane. What is the area of the hexagon?



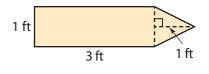


8. A field is shaped like the figure shown. What is the area of the field? Use 3.14 for π .

Class



- **9.** A bookmark is shaped like a rectangle with a semicircle attached at both ends. The rectangle is 12 cm long and 4 cm wide. The diameter of each semicircle is the width of the rectangle. What is the area of the bookmark? Use 3.14 for π .
- **10.** Multistep Alex is making 12 pennants for the school fair. The pattern he is using to make the pennants is shown in the figure. The fabric for the pennants costs \$1.25 per square foot. How much will it cost Alex to make 12 pennants?



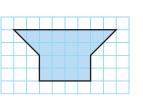
11. Reasoning A composite figure is formed by combining a square and a triangle. Its total area is 32.5 ft². The area of the triangle is 7.5 ft². What is the length of each side of the square? Explain.

Date.

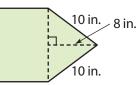


12. Represent Real-World Problems Christina plotted the shape of her garden on graph paper. She estimates that she will get about 15 carrots from each square unit. She plans to use the entire garden for carrots. About how many carrots can she expect to grow? Explain.

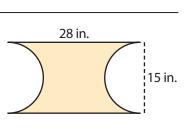
HOT



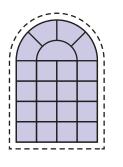
13. Analyze Relationships The figure shown is made up of a triangle and a square. The perimeter of the figure is 56 inches. What is the area of the figure? Explain.



14. Critical Thinking The pattern for a scarf is shown at right. What is the area of the scarf? Use 3.14 for π .



15. Persevere in Problem Solving The design for the palladium window shown includes a semicircular shape at the top. The bottom is formed by squares of equal size. A shade for the window will extend 4 inches beyond the perimeter of the window, shown by the dashed line around the window. Each square in the window has an area of 100 in².

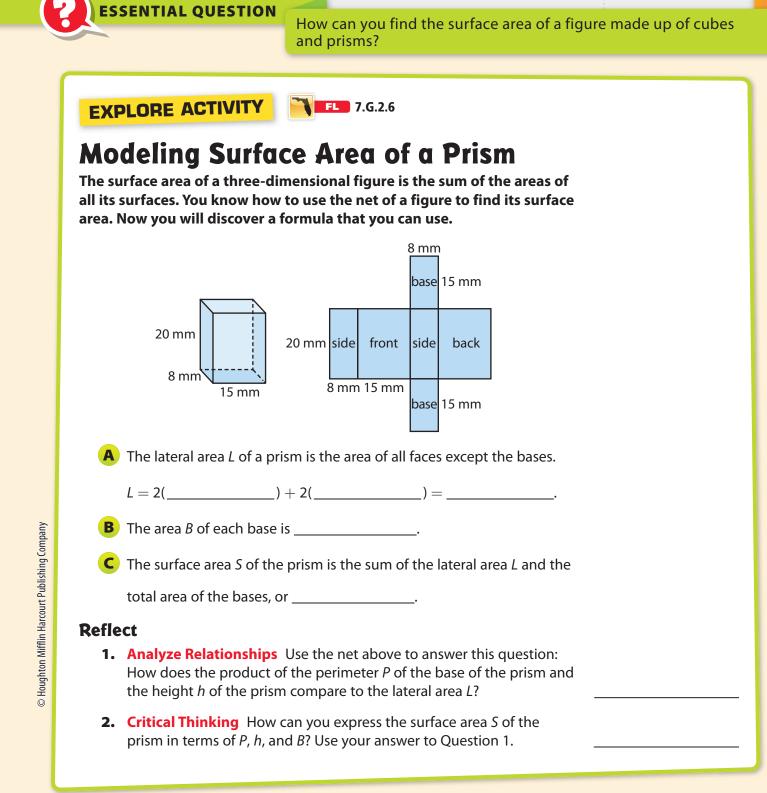


- **a.** What is the area of the window? Use 3.14 for π .
- **b.** What is the area of the shade? Round your answer to the nearest whole number.

Solving Surface Area Problems



Solve real-world and mathematical problems involving . . . surface area of ... three-dimensional objects composed of ... cubes and right prisms.





Finding the Surface Area of a Prism

Given a prism's dimensions, you can use a formula to find the surface area.

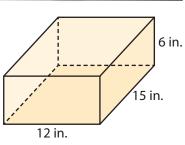
Surface Area of a Prism

The surface area S of a prism with base perimeter *P*, height *h*, and base area *B* is S = Ph + 2B.

EXAMPLE 1 🎇

My Notes

Erin is making a jewelry box of wood in the shape of a rectangular prism. The jewelry box will have the dimensions shown. She plans to spray paint the exterior of the box. How many square inches will she have to paint?



7.G.2.6

STEP 1

diagram helps you understand and solve the problem.

Make a sketch of the box. Drawing a

STEP 2

Identify a base, and find its area and perimeter.

Any pair of opposite faces can be the bases. For example, you can choose the bottom and top of the box as the bases.

$B = \ell \times w$	P = 2(12) + 2(15)	
= 12 × 15	= 24 + 30	
= 180 square inches	= 54 inches	

STEP 3

Identify the height, and find the surface area.

The height h of the prism is 6 inches. Use the formula to find the surface area.

$$S = Ph + 2B$$

S = 54(6) + 2(180) = 684 square inches

Erin will have to spray paint 684 square inches of wood.

YOUR TURN

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Mathematical Practices How can you express the formula for the surface area S of a rectangular prism in

terms of its dimensions *ℓ*, *w*, and *h*?

3. A brand of uncooked spaghetti comes in a box that is a rectangular prism with a length of 9 inches, a width of 2 inches, and a height of $1\frac{1}{2}$ inches.

What is the surface area of the box? _____

Finding the Surface Area of a Composite Solid

A composite solid is made up of two or more solid figures. To find the surface area of a composite solid, find the surface area of each figure. Subtract any area not on the surface.

8 cm

30 cm

EXAMPLE 2 Problem

Daniel built the birdhouse shown. What was the surface area of the birdhouse before the hole was drilled?

Analyze Information

Identify the important information.

- The top is a triangular prism with h = 24 cm. The base is a triangle with height 8 cm and base 30 cm.
- The bottom is a rectangular prism with *h* = 18 cm. The base is a 30 cm by 24 cm rectangle.
- One face of each prism is not on the surface of the figure.

Formulate a Plan

Find the surface area of each prism.

Add the areas. Subtract the areas of the parts not on the surface.

Solve

Find the area of the triangular prism.

Perimeter = 17 + 17 + 30 = 64 cm; Base area = $\frac{1}{2}$ (30)(8) = 120 cm²

Surface area = Ph + 2B

 $= 64(24) + 2(120) = 1,776 \text{ cm}^2$

Find the area of the rectangular prism.

Perimeter = 2(30) + 2(24) = 108 cm; Base area = 30(24) = 720 cm²

Surface area = Ph + 2B

= 108(18) + 2(720) = 3,384 cm²

Add. Then subtract twice the areas of the parts not on the surface.

Surface area = $1,776 + 3,384 - 2(720) = 3,720 \text{ cm}^2$

The surface area before the hole was drilled was 3,720 cm².

La Justify and Evaluate

You can check your work by using a net to find the surface areas.



17 cm

18 cm

, 24 cm





Math Talk Mathematical Practices

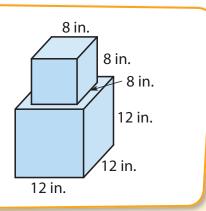
How could you find the surface area by letting the front and back of the prism be the bases?



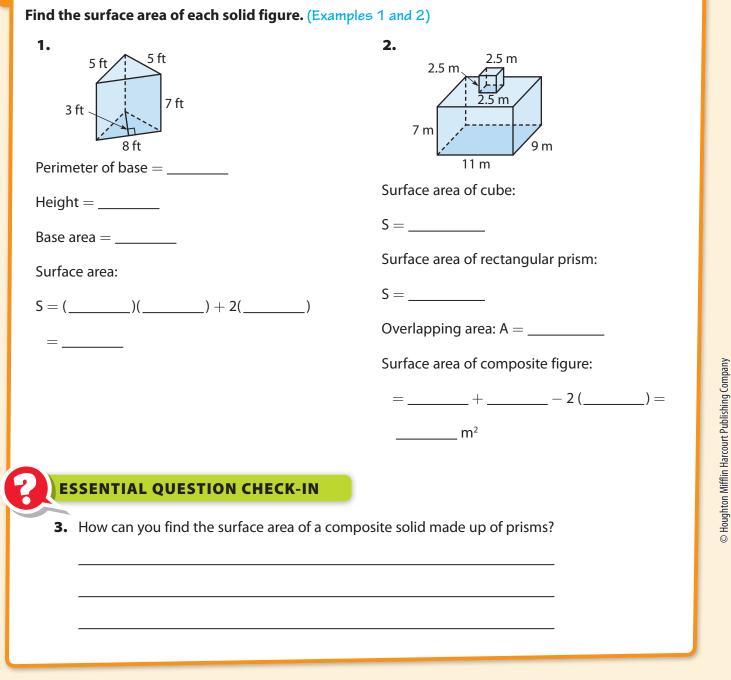


4. Dara is building a plant stand. She wants to stain the plant stand, except for the bottom of the larger prism. Find the surface area of the part of the plant

stand she will stain.



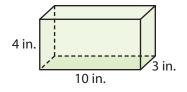
Guided Practice



9.4 Independent Practice



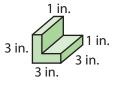
4. Carla is wrapping a present in the box shown. How much wrapping paper does she need, not including overlap?



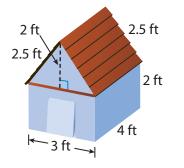
 Dmitri wants to cover the top and sides of the box shown with glass tiles that are 5 mm square. How many tiles does he need?

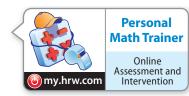


6. Shera is building a cabinet. She is making wooden braces for the corners of the cabinet. Find the surface area of each brace.



7. The doghouse shown has a floor, but no windows. Find the total surface area of the doghouse, including the door.

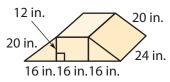




Date

Eddie built the ramp shown to train his puppy to do tricks. Use the figure for 8–9.

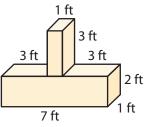
Class



8. Analyze Relationships Describe two ways to find the surface area of the ramp.

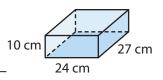
9. What is the surface area of the ramp?

Marco and Elaine are building a stand like the one shown to display trophies. Use the figure for 10–11.



- **10.** What is the surface area of the stand?
- 11. Critique Reasoning Marco and Elaine want to paint the entire stand silver. A can of paint covers 25 square feet and costs \$6.79. They set aside \$15 for paint. Is that enough? Explain.

12. Henry wants to cover the box shown with paper without any overlap. How many square centimeters will be covered with paper?



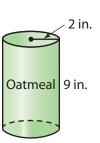
13. What If? Suppose the length and width of the box in Exercise 12 double. Does the surface area *S* double? Explain.



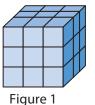
FOCUS ON HIGHER ORDER THINKING

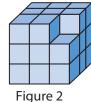
14. Persevere in Problem Solving Enya is building a storage cupboard in the shape of a rectangular prism. The rectangular prism has a square base with side lengths of 2.5 feet and a height of 3.5 feet. Compare the amount of paint she would use to paint all but the bottom surface of the prism to the amount she would use to paint the entire prism.

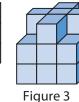
15. Interpret the Answer The oatmeal box shown is shaped like a cylinder. Use a net to find the surface area *S* of the oatmeal box to the nearest tenth. Then find the number of square feet of cardboard needed for 1,500 oatmeal boxes. Round your answer to the nearest whole number.

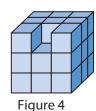


16. Analyze Relationships A prism is made of centimeter cubes. How can you find the surface area of the prism in Figure 1 without using a net or a formula? How does the surface area change in Figures 2, 3, and 4? Explain.







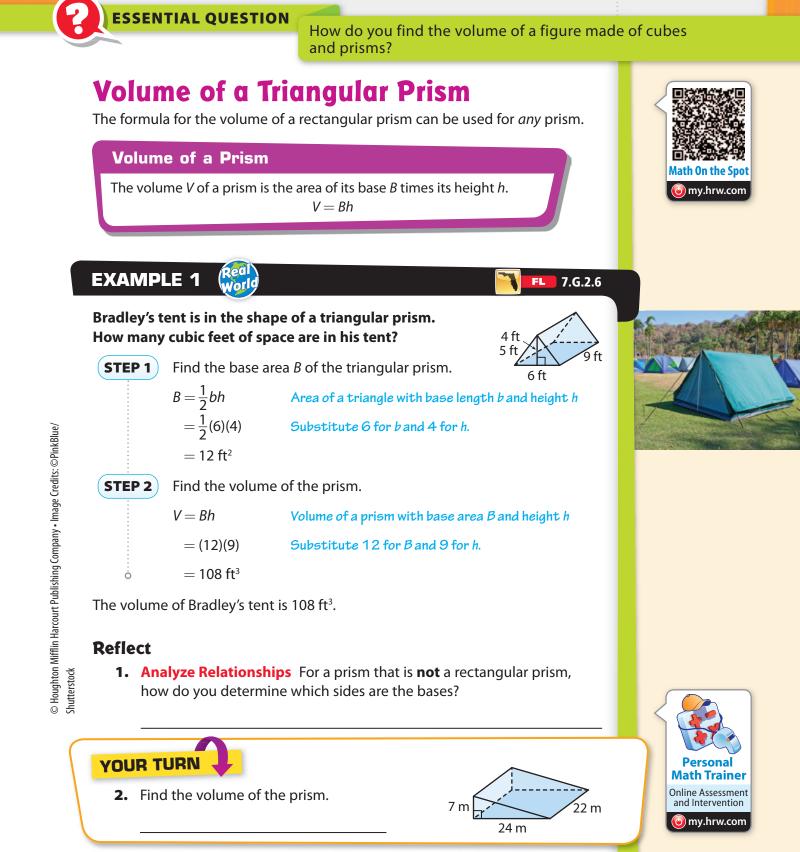


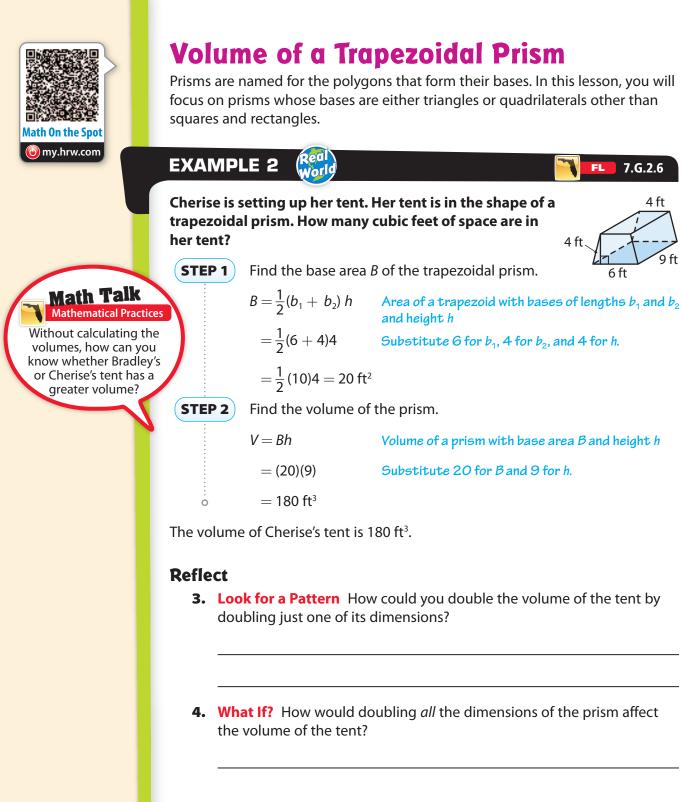
Work Area

Solving Volume 9.5 Problems



Solve real-world and mathematical problems involving ... volume of ... three-dimensional objects composed of ... cubes, and right prisms.







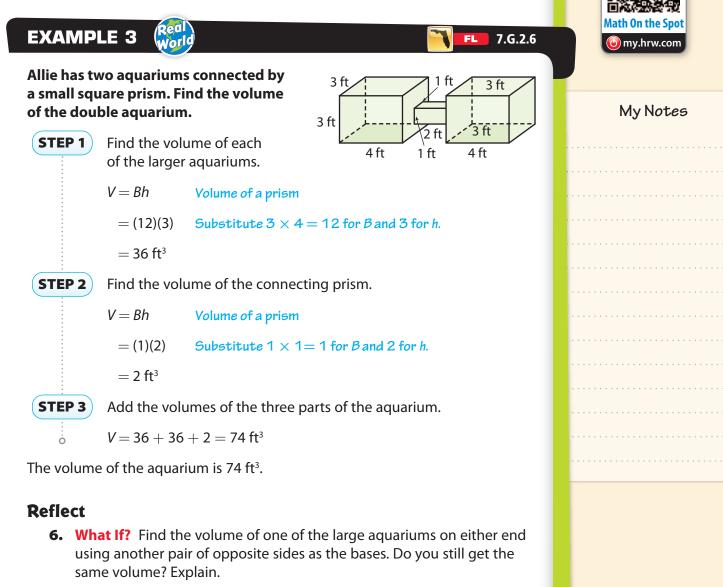


5. Find the volume of the prism.

12 cm 10 cm -8 cm

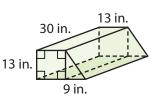
Volume of a Composite Solid

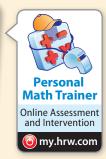
You can use the formula for the volume of a prism to find the volume of a composite figure that is made up of prisms.





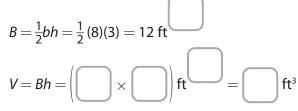
7. The figure is composed of a rectangular prism and a triangular prism. Find the volume of the figure.





Guided Practice

1. Find the volume of the triangular prism. (Example 1)



2. Find the volume of the trapezoidal prism. (Example 2)

$$B = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(15 + 5)(3) = 30m$$

$$V = Bh = \left(\bigcirc \times \bigcirc \right) m \bigcirc = \bigcirc m^3$$

3. Find the volume of the composite figure. (Example 3)

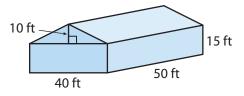
Volume of rectangular prism = _____

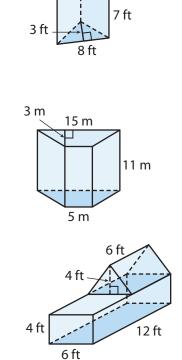
Volume of triangular prism = _____

Volume of composite figure = _____

Find the volume of each figure. (Examples 2 and 3)

4. The figure shows a barn that Mr. Fowler is building for his farm.

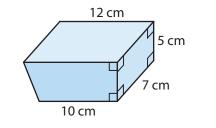




5 ft

5 ft

5. The figure shows a container, in the shape of a trapezoidal prism, that Pete filled with sand.



ESSENTIAL QUESTION CHECK-IN

6. How do you find the volume of a composite solid formed by two or more prisms?

12.

 $B \approx 23.4 \text{ in}^2$

9.5 Independent Practice

FL 7.G.2.6

Name

- **7.** A trap for insects is in the shape of a triangular prism. The area of the base is 3.5 in² and the height of the prism is 5 in. What is the volume of this trap?
- **8.** Arletta built a cardboard ramp for her little brothers' toy cars. Identify the shape of the ramp. Then find its volume.
- 9. Alex made a sketch for a homemade soccer goal he plans to build. The goal will be in the shape of a triangular prism. The legs of the right triangles at the sides of his goal measure 4 ft and 8 ft, and the opening along the front is 24 ft. How much space is contained within this goal?
- **10.** A gift box is in the shape of a trapezoidal prism with base lengths of 7 inches and 5 inches and a height of 4 inches. The height of the gift box is 8 inches. What is the volume of the gift box?
- **11. Explain the Error** A student wrote this statement: "A triangular prism has a height of 15 inches and a base area of 20 square inches. The volume of the prism is 300 square inches." Identify and correct the error.

13.

3.75 n

Find the volume of each figure. Round to the nearest hundredth if necessary.

3 in.

3 in.

3 in. 3 in.

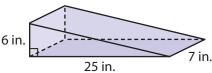
14. Multi-Step Josie has 260 cubic centimeters of candle wax. She wants to make a hexagonal prism candle with a base area of 21 square centimeters and a height of 8 centimeters. She also wants to make a triangular prism candle with a height of 14 centimeters. Can the base area of the triangular prism candle be 7 square centimeters? Explain.



Date







7.5 m

15 m

7.5 m

3.75 m



Class

15. A movie theater offers popcorn in two different containers for the same price. One container is a trapezoidal prism with a base area of 36 square inches and a height of 5 inches. The other container is a triangular prism with a base area of 32 square inches and a height of 6 inches. Which container is the better deal? Explain.

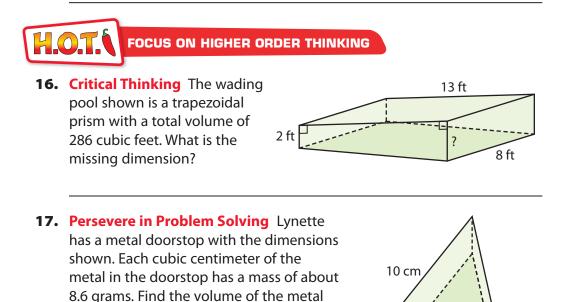


2.5 cm

8 cm

6 cm

Work Area



18. Analyze Relationships What effect would tripling all the dimensions of a triangular prism have on the volume of the prism? Explain your reasoning.

in the doorstop. Then find the mass of

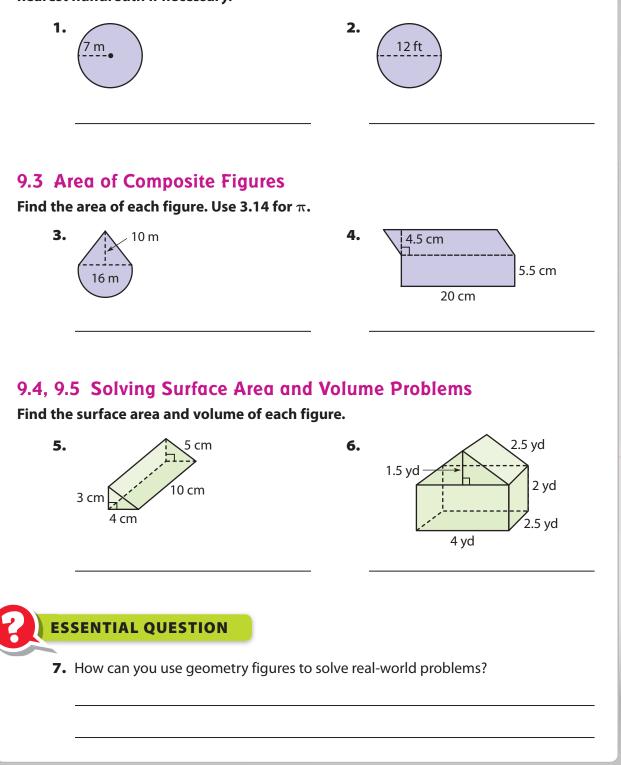
the doorstop.

19. Persevere in Problem Solving Each of two trapezoidal prisms has a volume of 120 cubic centimeters. The prisms have no dimensions in common. Give possible dimensions for each prism.



9.1, 9.2 Circumference and Area of Circles

Find the circumference and area of each circle. Use 3.14 for π . Round to the nearest hundredth if necessary.



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MODULE 9 MIXED REVIEW Assessment Readiness

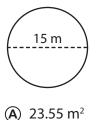


Selected Response

1. What is the circumference of the circle?

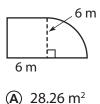


- (A) 34.54 m
- **B** 69.08 m
- **(C)** 379.94 m
- **D** 1,519.76 m
- 2. What is the area of the circle?



(B) 47.1 m²

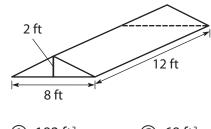
- (C) 176.625 m²
 (D) 706.5 m²
- 3. What is the area of the figure?



(B) 36 m²

- C 64.26 m²
 D 92.52 m²
- **4.** A one-year membership to a health club costs \$480. This includes a \$150 fee for new members that is paid when joining. Which equation represents the monthly cost *x* in dollars for a new member?
 - (A) 12x + 150 = 480
 - **B** $\frac{x}{12} + 150 = 480$
 - (C) 12x + 480 = 150
 - (D) $\frac{x}{12} + 480 = 150$

5. What is the volume of the prism?



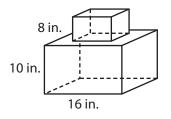
- (A) 192 ft³
 (C) 69 ft³
 (B) 48 ft³
 (D) 96 ft³
- 6. A school snack bar sells a mix of granola and raisins. The mix includes 2 pounds of granola for every 3 pounds of raisins. How many pounds of granola are needed for a

mix that includes 24 pounds of raisins?

- (A) 16 pounds (C) 48 pounds
- (B) 36 pounds (D) 120 pounds
- **7.** Find the percent change from \$20 to \$25.
 - (A) 25% decrease (C) 20% decrease
 - (B) 25% increase (D) 20% increase

Mini-Task

8. Each dimension of the smaller prism is half the corresponding dimension of the larger prism.



- a. What is the surface area of the figure?
- **b.** What is the volume of the figure?

Study Guide Review



Modeling Geometric Figures

ESSENTIAL QUESTION

How can you apply geometry concepts to solve real-world problems?

EXAMPLE 1

Use the scale drawing to find the perimeter of Tim's yard.



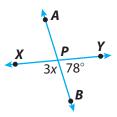
2 cm : 14 ft

$\frac{2 \text{ cm}}{14 \text{ ft}} = \frac{1 \text{ cm}}{7 \text{ ft}}$	1 cm in the drawing equals 7 feet in the actual yard.
$\frac{1\mathrm{cm}\times15}{7\mathrm{ft}\times15} = \frac{15\mathrm{cm}}{105\mathrm{ft}}$	15 cm in the drawing equals 105 feet in the actual yard. Tim's yard is 105 feet long.
$\frac{1\mathrm{cm}\times4}{7\mathrm{ft}\times4} = \frac{4\mathrm{cm}}{28\mathrm{ft}}$	4 cm in the drawing equals 7 feet in the actual yard. Tim's yard is 28 feet wide.

Perimeter is twice the sum of the length and the width. So the perimeter of Tim's yard is 2(105 + 28) = 2(133), or 266 feet.

EXAMPLE 2

Find (a) the value of x and (b) the measure of $\angle APY$.



a. $\angle XPB$ and $\angle YPB$ are supplementary.

 $3x + 78^{\circ} = 180^{\circ}$

 $3x = 102^{\circ}$

$$x = 34^{\circ}$$

b. $\angle APY$ and $\angle XPB$ are vertical angles.

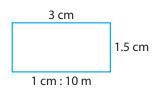
 $m \angle APY = m \angle XPB = 3x = 102^{\circ}$

Key Vocabulary

adjacent angles (ángulos adyacentes) complementary angles (ángulos complementarios) congruent angles (ángulos congruentes) cross section (sección transversal) intersection (intersección) scale (escala) scale drawing (dibujo a escala) supplementary angles (ángulos suplementarios) vertical angles (ángulos opuestos por el vértice)

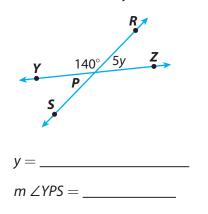
EXERCISES

In the scale drawing of a park, the scale is
 1 cm: 10 m. Find the area of the actual park.

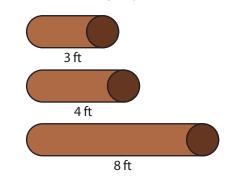


(Lesson 8.1)

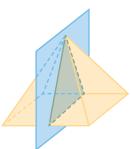
2. Find the value of y and the measure of $\angle YPS$ (Lesson 8.4)



3. Kanye wants to make a triangular flower bed using logs with the lengths shown below to form the border. Can Kanye form a triangle with the logs without cutting any of them? Explain. (Lesson 8.2)



4. In shop class, Adriana makes a pyramid with a 4-inch square base and a height of 6 inches. She then cuts the pyramid vertically in half as shown. What is the area of each cut surface? (Lesson 8.3)





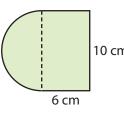
Circumference, Area, and Volume

ESSENTIAL QUESTION

How can you use geometry concepts to solve real-world problems?

EXAMPLE 1

Find the area of the composite figure. It consists of a semicircle and a rectangle.



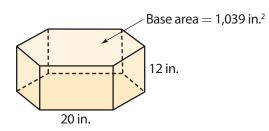
Area of semicircle = $0.5(\pi r^2)$ $\approx 0.5(3.14)25$ $\approx 39.25 \text{ cm}^2$ Area of rectangle = ℓw = 10(6)

 $= 60 \text{ cm}^2$

The area of the composite figure is approximately 99.25 square centimeters.

EXAMPLE 2

Find the volume and surface area of the regular hexagonal prism hat box shown. Each side of the hexagonal base is 20 inches.



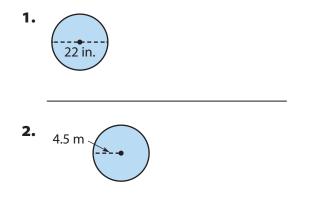
Use the formulas for volume and surface area of a prism.

V = Bh S = Ph + 2B = 1,039(12) $= 12,468 \text{ in}^{3}$ S = Ph + 2B = 120(12) + 2(1,039) = 1,440 + 2,078 $= 3,518 \text{ in}^{2}$

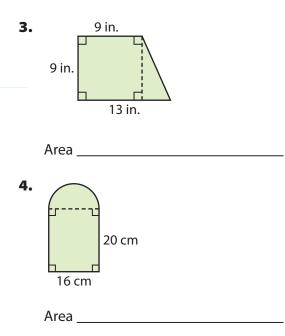
Key Vocabulary

circumference (circunferencia) composite figure (figura compuesta) diameter (diámetro) radius (radio)

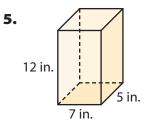
Find the circumference and area of each circle. Round to the nearest hundredth. (Lessons 9.1, 9.2)



Find the area of each composite figure. Round to the nearest hundredth if necessary. (Lesson 9.3)



Find the volume of each figure. (Lesson 9.5)



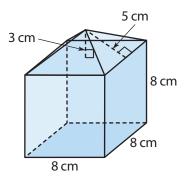
6. The volume of a triangular prism is 264 cubic feet. The area of a base of the prism is 48 square feet. Find the height of the prism.

(Lesson 9.5)

300 Unit 4

EXERCISES

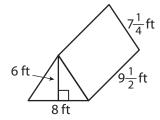
A glass paperweight has a composite shape: a square pyramid fitting exactly on top of an 8 centimeter cube. The pyramid has a height of 3 cm. Each triangular face has a height of 5 centimeters. (Lessons 9.4, 9.5)



- 7. What is the volume of the paperweight? _____
- 8. What is the total surface area of the paperweight? _____

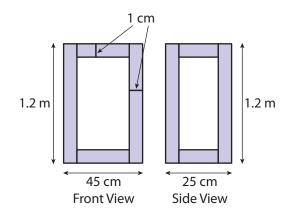
Unit 4 Performance Tasks

1. CAREERS IN MATH Product Design Engineer Miranda is a product design engineer working for a sporting goods company. She designs a tent in the shape of a triangular prism. The approximate dimensions of the tent are shown in the diagram.



- **a.** How many square feet of material does Miranda need to make the tent (including the floor)? Show your work.
- **b.** What is the volume of the tent? Show your work.
- **c.** Suppose Miranda wants to increase the volume of the tent by 10%. The specifications for the height (6 feet) and the width (8 feet) must stay the same. How can Miranda meet this new requirement? Explain.

- 2. Li is making a stand to display a sculpture made in art class. The stand will be 45 centimeters wide, 25 centimeters long, and 1.2 meters high.
 - **a.** What is the volume of the stand? Write your answer in cubic centimeters.
 - b. Li needs to fill the stand with sand so that it is heavy and stable. Each piece of wood is 1 centimeter thick. The boards are put together as shown in the figure, which is not drawn to scale. How many cubic centimeters of sand does she need to fill the stand? Explain how you found your answer.

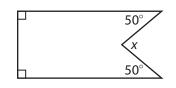




Assessment Readiness

Selected Response

1. A school flag is in the shape of a rectangle with a triangle removed as shown.



What is the measure of angle *x*?

- **(A)** 50° **(C)** 90°
- **B** 80° **D** 100°
- 2. On a map with a scale of 2 cm = 1 km, the distance from Beau's house to the beach is
 4.6 centimeters. What is the actual distance?

(A) 2.3 km	🔘 6.5 km
B 4.6 km	Ď 9.2 km

3. Lalasa and Yasmin are designing a triangular banner to hang in the school gymnasium. They first draw the design on paper. The triangle has a base of 5 inches and a height of 7 inches. If 1 inch on the drawing is equivalent to 1.5 feet on the actual banner, what will the area of the actual banner be?

- (B) 52.5 ft² (D) 78.75 ft²
- **4.** Sonya has four straws of different lengths: 2 cm, 8 cm, 14 cm, and 16 cm. How many triangles can she make using the straws?
 - A no triangle
 - (B) one triangle
 - © two triangles
 - D more than two triangles

5. A one-topping pizza costs \$15.00. Each additional topping costs \$1.25. Let *x* be the number of additional toppings. You have \$20 to spend. Which equation can you solve to find the number of additional toppings you can get on your pizza?

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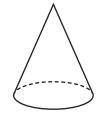
Personal Math Trainer

Online Assessment and Intervention

- (A) 15x + 1.25 = 20
- **B** 1.25x + 15 = 20
- (c) 15x 1.25 = 20
- **(D)** 1.25x 15 = 20
- 6. A bank offers a home improvement loan with simple interest at an annual rate of 12%. J.T. borrows \$14,000 over a period of 3 years. How much will he pay back altogether?

(A) \$15,680	© \$19,040
B \$17,360	D \$20,720

- **7.** What is the volume of a triangular prism that is 75 centimeters long and that has a base with an area of 30 square centimeters?
 - A 2.5 cubic centimeters
 - (B) 750 cubic centimeters
 - C 1,125 cubic centimeters
 - D 2,250 cubic centimeters
- **8.** Consider the right circular cone shown.



If a vertical plane slices through the cone to create two identical half cones, what is the shape of the cross section?

(A) a rectangle (C) a triangle

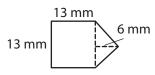
B a square

D a circle

9. The radius of the circle is given in meters. What is the circumference of the circle? Use 3.14 for π .



- (A) 25.12 meters
- **B** 50.24 meters
- © 200.96 meters
- **D** 803.84 meters
- **10.** The dimensions of the figure are given in millimeters. What is the area of the two-dimensional figure?



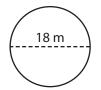
- (A) 39 square millimeters
- (B) 169 square millimeters
- © 208 square millimeters
- D 247 square millimeters
- **11.** A forest ranger wants to determine the radius of the trunk of a tree. She measures the circumference to be 8.6 feet. What is the trunk's radius to the nearest tenth of a foot?
 - (A) 1.4 ft (B) 4.3 ft
 - © 2.7 ft D 17.2 ft



It is helpful to draw or redraw a figure. Answers to geometry problems may become clearer as you redraw the figure.

- **12.** What is the measure in degrees of an angle that is supplementary to a 74° angle?
 - **(A)** 16°
 - **B** 74°
 - C 90°
 - **D** 106°

- **13.** What is the volume in cubic centimeters of a rectangular prism that has a length of 6.2 centimeters, a width of 3.5 centimeters, and a height of 10 centimeters?
 - (A) 19.7 cm³ (C) 217.0 cm³
 - **(B)** 108.5 cm³ **(D)** 237.4 cm³
- **14.** A patio is the shape of a circle with diameter shown.



What is the area of the patio? Use 3.14 for π .

- (A) 9.00 m²
- **B** 28.26 m²
- C 254.34 m²
- **D** 1,017.36 m²

Mini-Tasks

- **15.** Petra fills a small cardboard box with sand. The dimensions of the box are 3 inches by 4 inches by 2 inches.
 - **a.** What is the volume of the box?
 - **b.** Petra decides to cover the box by gluing on wrapping paper. How much wrapping paper does she need to cover all six sides of the box?
 - c. Petra has a second, larger box that is 6 inches by 8 inches by 4 inches. How many times larger is the volume of this second box? The surface area?